A NOVEL HESITANT INTUITIONISTIC FUZZY LINGUISTIC AHP METHOD AND ITS APPLICATION TO PRIORITIZATION OF INVESTMENT ALTERNATIVES

Ali Karasan
Yildiz Technical University
Turkey
akarasan@yildiz.edu.tr

ABSTRACT

Intuitionistic fuzzy extensions are the most used type of fuzzy extension in the literature because they better represent the decision makers' strength of commitment on the considered subject in an effective way including membership and non-membership functions. On the other hand, decision makers may assign more than one intuitionistic fuzzy number when they are hesitant about assigning a membership degree and a non-membership degree. Hesitant fuzzy sets, another extension of ordinary fuzzy sets, help decision makers assign different values to the same element in an attempt to reflect the decision makers' hesitation. Utilizing these two types of fuzzy sets captures both the uncertainty and ambiguity of the considered problem and helps eliminate the weaknesses of each fuzzy extension. In this study, hesitant intuitionistic fuzzy linguistic sets (HIFLSs) are used to extend the Analytic Hierarchy Process (AHP). The developed method is applied to an investment prioritization problem based on relevant risk factors. Comparative analyses with intuitionistic fuzzy AHP and hesitant fuzzy AHP methods are conducted in order to validate the proposed method. A sensitivity analysis is also applied to present the stability of the results of the hesitant intuitionistic fuzzy linguistic AHP method.

Keywords: Hesitant fuzzy sets; intuitionistic fuzzy sets; AHP; prioritization; hesitant intuitionistic fuzzy linguistic sets

1. Introduction

Multi-criteria decision making (MCDM) methods are one of the most used decision-making tools in the literature to deal with sets of criteria and alternatives simultaneously. The Analytic Hierarchy Process (AHP), which is one of the most useful multi-criteria decision-making methods that was originally developed by Saaty, aims to achieve the weighting of values of independent inputs by comparing them in a pairwise comparison matrix (Saaty, 1980). These inputs can be quantitative such as cost, profit, and distance and/or qualitative such as judgments, behaviors, and preferences. Before the evaluation process starts, the values of the compared inputs must be checked to calculate the matrix's consistency. Since the inconsistencies are in the decision makers' decisions, the

AHP method is tolerated up to a lower limit of consistency ratio which is 90%. But, it is difficult to construct consistent matrices in systems where information is uncertain and decision makers are hesitant. Therefore, the classic AHP method cannot provide effective results and needs some extensions to handle these deficiencies.

Fuzzy logic is one of the most advantageous ways of handling uncertainty by representing it with the degree of membership function and was introduced by Zadeh (1965). It is an invaluable way of finding efficient solutions for researchers in the fields of engineering, mathematics, medical sciences, computer sciences, natural and applied sciences, business analysis, public relations, and human behaviors. In order to increase this efficiency, the fuzzy sets are extended with many types in light of fuzzy logic. These types are as follows: Type-2 fuzzy sets, intuitionistic fuzzy sets, neutrosophic fuzzy sets, hesitant fuzzy sets, Pythagorean fuzzy sets, and orthopair fuzzy sets (Zadeh, 1975; Atanassov, 1986; Smarandache, 1995; Torra, 2010; Yager & Abbasov, 2013; Ciucci, 2016). Besides, as many extensions of fuzzy sets have been introduced, many multi-criteria decision-making methods that are related to fuzzy logic have kept pace with them. The AHP method has been extended in many forms including Buckley's fuzzy AHP method, Chang's fuzzy AHP method, intuitionistic fuzzy AHP method, type-2 fuzzy AHP method, hesitant fuzzy AHP method, neutrosophic AHP method, and the Pythagorean fuzzy AHP method (Buckley, 1985; Chang, 1996; Wu et al., 2013; Kahraman et al., 2014; Öztaysi et al., 2015; Abdel-Basset et al., 2017; Ilbahar et al., 2018). Also, Ozdemir & Sahin (2018), Durao et al. (2018), Pipatprapa et al. (2018), Yucesan & Kahraman (2019), Vladeanu & Matthews (2019), Vyas et al. (2019), Xu et al. (2019), Arulbalaji et al. (2019) are some studies that applied the AHP method with its extension to different areas. Pipatprapa et al. (2018) presented an environmental performance assessment for the food industry in Thailand by using the AHP method. Durao et al. (2018) presented a process selection for the internet of things by using the AHP method. Ozdemir & Sahin (2018) studied the AHP method for the selection of a location for a solar photovoltaic power plant. Arulbalaji et al. (2019) studied geographical information systems and AHP techniques for the delineation of potential groundwater zones in India. Xu et al. (2019) studied an entropy weight modified AHP hierarchy model for the construction of a regional informatization ecological environment. Vyas et al. (2019) developed a green building rating system by using an extended AHP model with fuzzy integrals. Vladeanu & Matthews (2019) presented a risk based asset management study for wastewater pipes by using the AHP method. Yucesan & Kahraman (2019) studied risk evaluation and prevention in a hydropower plant operation by using the Pythagorean fuzzy AHP method.

Intuitionistic fuzzy sets are the most used type of fuzzy extensions in the literature since they have the ability to better represent the decision makers' strength of commitment on the considered subject including membership and non-membership functions simultaneously. On the other hand, decision makers may assign more than one intuitionistic fuzzy number when they are hesitant in assigning a membership degree and a non-membership degree. Hesitant fuzzy sets, another extension of ordinary fuzzy sets, help decision makers assign different values to the same element in order to reflect the decision makers' hesitation. Utilizing these two types of fuzzy sets captures both the uncertainty and ambiguity of the considered problem and helps eliminate the weaknesses of each fuzzy extension.

ISSN 1936-6744
https://doi.org/10.13033/ijahp.v11i1.610

This paper contributes to the literature as follows:

- The proposed method has the ability to reflect both pessimistic and optimistic judgements of the decision maker by using intuitionistic fuzzy sets.
- Since intuitionistic fuzzy sets offer a larger domain than ordinary fuzzy sets for the decision makers to introduce their judgements, the proposed method enables the representation of the decision makers' hesitancy by using both membership and non-membership functions of an element to a set.
- The proposed method can be a supportive decision making tool for the decision makers who intend to work with MCDM methods with uncertain information.

In investment decision making problems, managers aim to minimize adverse effects of risks by using personal assessments, and experiences and mathematical models when they are deciding the appropriate alternatives to invest in. In this paper, we propose a novel hesitant intuitionistic fuzzy linguistic AHP method for the prioritization of investment alternatives to help make the best decision.

The paper is organized as follows: in Section 2, the steps of our proposed methodology and details of hesitant and intuitionistic fuzzy sets are presented. In Section 3, an application of a company that wants to invest to build a factory is demonstrated. The paper ends with a conclusion and suggestions for further research.

2. Methodology

In this section, the preliminaries of hesitant fuzzy sets and intuitionistic fuzzy sets, and a novel hesitant intuitionistic fuzzy linguistic AHP method are given, respectively.

2.1 Hesitant fuzzy sets

Hesitant fuzzy sets (HFS), initially developed by Torra (2010), are the extensions of regular fuzzy sets which handle the situations where a set of values are possible for the membership of a single element (Rodrigez et al., 2012). Torra (2010) defines hesitant fuzzy sets as follows. Let *X* be a fixed set, a hesitant fuzzy set (HFS) on *X* is in terms of a function that when applied to *X* returns a subset of [0, 1]. A mathematical expression for HFS is as follows:

$$E = \{< x, h_E(x) > \mid x \epsilon X \} \quad (1)$$

where $h_E(x)$ is a set of some values in [0, 1], denoting the possible membership degrees of the element $x \epsilon X$ to the set *E*. Xu & Xia (2011) call $h = h_E(x)$ a hesitant fuzzy element (HFE).

Some basic definitions about hesitant sets are given as follows (Torra, 2010):

$$\lambda h = \cup_{\gamma \in h} \{1 - (1 - \gamma)^{\lambda}\} \quad (2)$$

$$h_1 \oplus h_2 = \cup_{\gamma_1 \in h_1, \gamma_2 \in h_2,} \{\gamma_1 + \gamma_2 - \gamma_1 \gamma_2\} \quad (3)$$

ISSN 1936-6744
https://doi.org/10.13033/ijahp.v11i1.610

$$h_1 \otimes h_2 = \cup_{\gamma_1 \in h_1, \gamma_2 \in h_2,} \{ \gamma_1 \gamma_2 \} \quad (4)$$

where h_1, h_2 are HFEs and l is the number of elements in an HFE, which is called length.

2.2 Intuitionistic fuzzy sets

Intuitionistic fuzzy sets (IFSs) were introduced by Atanassov (1986). An IFS includes two membership values named membership and non-membership for describing any x in X such that their sum is at most equal to 1, which is demonstrated in Definition 1 (Atanassov, 1986).

Definition 1. Let $X \neq \emptyset$ be a given set. An intuitionistic fuzzy set in X is an object A which is called an intuitionistic fuzzy number (IFN), if it holds the following conditions:

$$\tilde{A} = \{\langle x, \mu_{\tilde{A}}(x), v_{\tilde{A}}(x) \rangle; x \in X\} \quad (5)$$

where $\mu_{\tilde{A}}(x): X \rightarrow [0,1]$ and $v_{\tilde{A}}(x): X \rightarrow [0,1]$ satisfies the condition of

$$0 \leq \mu_{\tilde{A}}(x) + v_{\tilde{A}}(x) \leq 1, \text{ for every } x \in X. \quad (6)$$

Besides, $\pi_{\tilde{A}}(x) = 1 - \mu_{\tilde{A}}(x) + v_{\tilde{A}}(x)$ is the hesitation degree of element $\tilde{A}$ in set X. It is obvious that we obtain $0 \leq \pi_{\tilde{A}} \leq 1$ for every $x \in X$.

Definition 2. The operations of addition $\oplus$ and multiplication $\otimes$ on IFNs were defined by Atanassov (1986) as follows. Let $\tilde{A} = \langle \mu_A, v_A \rangle$ and $\tilde{B} = \langle \mu_B, v_B \rangle$ be IFNs.

$$\tilde{A} \oplus \tilde{B} = \langle \mu_A + \mu_B - \mu_A \mu_B, v_A v_B \rangle \quad (7)$$

$$\tilde{A} \otimes \tilde{B} = \langle \mu_A \mu_B, v_A + v_B - v_A v_B \rangle \quad (8)$$

Definition 3. Let $\tilde{A} = \langle \mu_A, v_A \rangle$ be IFN. The defuzzification operation of an intuitionistic fuzzy number is given as follows:

$$\mathfrak{H}_A = \frac{\mu_A + (1 - \vartheta_A)}{2} \quad (9)$$

2.3 Hesitant Intuitionistic Fuzzy Linguistic (HIFL) AHP method

Step 1. Construct the collected pairwise comparison matrices for criteria and alternatives of the expert's appraisals by using linguistic terms.

Step 2. The linguistic terms are transformed into IFNs using the scale given in Table 1. A 10% hesitancy exists in this scale.

Table 1
Linguistic scale for hesitant intuitionistic fuzzy linguistic (HIFL) AHP method

Linguistic Term	**IFN**
Certainly Low Importance - CLI	<[0.05, 0.85]>
Very Low Importance - VLI	<[0.2, 0.7]>
Low Importance - LI	<[0.35, 0.55]>
Equal Importance - EI	<[0.5, 0.5]>
High Importance - HI	<[0.55, 0.35]>
Very High Importance - VHI	<[0.7, 0.2]>
Certainly High Importance - CHI	<[0.85, 0.05]>

Step 3. Determine the minimum and maximum of the membership and non-membership values which are given in a collected pairwise comparison matrix.

Step 4. Calculate the optimistic and pessimistic weight values of the pairwise comparison matrix by using Equation 9.

Step 5. Calculate the mid points of the optimistic and pessimistic weight values.

Step 6. Normalize the mid points and rank them in descending order.

3. Illustrative example

A company in Turkey wants to be a distributor of the components of unmanned aerial vehicles for the defense industry sector. To make a distribution agreement for the Turkey market, the company performed preparatory work about the conditions of component producers. Through this work, the company determined four producers as the alternatives which are suitable for the budget. It was the consensus of the company to assess these alternatives with respect to the four criteria given below.

- Investment cost
- Maintenance cost
- Average life expectancy of the components
- Flexibility of the components to the different models

3.1 Steps of the method

To check the applicability of the proposed method, the hesitant intuitionistic fuzzy linguistic (HIFL) AHP method was applied as follows:

The collected pairwise comparison matrices are given in Table 2.

ISSN 1936-6744
https://doi.org/10.13033/ijahp.v11i1.610

Table 2
Collected pairwise comparison matrices

wrt Goal	C1			C2			C3			C4		
C1	EI	EI	EI	LI	EI	-	EI	-	LI	HI	-	-
C2	HI	EI	-	EI	EI	EI	LI	EI	VLI	VHI	HI	-
C3	EI	-	HI	HI	EI	VHI	EI	EI	EI	VHI	CHI	-
C4	LI	-	-	VLI	LI	-	VLI	CLI	-	EI	EI	EI
wrt C1	**AL1**			**AL2**			**AL3**			**AL4**		
AL1	EI	EI	EI	LI	EI	-	EI	-	LI	HI	-	-
AL2	HI	EI	-	EI	EI	EI	LI	EI	VLI	VHI	HI	-
AL3	EI	-	LI	HI	EI	VHI	EI	EI	EI	VHI	CHI	-
AL4	LI	-	-	VLI	LI	-	VLI	CLI	-	EI	EI	EI
wrt C2	**AL1**			**AL2**			**AL3**			**AL4**		
AL1	EI	EI	EI	LI	HI	-	HI	-	-	EI	HI	-
AL2	HI	LI	-	EI	EI	EI	VLI	VHI	-	HI	EI	LI
AL3	LI	-	-	VHI	VLI	-	EI	EI	EI	VLI	LI	-
AL4	EI	LI	-	LI	EI	HI	VHI	HI	-	EI	EI	EI
wrt C3	**AL1**			**AL2**			**AL3**			**AL4**		
AL1	EI	EI	EI	LI	HI	VHI	HI	LI	-	EI	HI	-
AL2	HI	LI	VLI	EI	EI	EI	VLI	HI	-	HI	EI	LI
AL3	LI	HI	-	VHI	LI	-	EI	EI	EI	VLI	LI	-
AL4	EI	LI	-	LI	EI	HI	VHI	HI	-	EI	EI	EI
wrt C4	**AL1**			**AL2**			**AL3**			**AL4**		
AL1	EI	EI	EI	LI	HI	-	HI	EI	-	EI	HI	HI
AL2	HI	LI	-	EI	EI	EI	VLI	LI	HI	HI	EI	LI
AL3	LI	EI	-	VHI	HI	LI	EI	EI	EI	HI	LI	LI
AL4	EI	LI	LI	LI	EI	HI	LI	HI	HI	EI	EI	EI
wrt: with respect to												

To clarify Table 2, the comparison of *C2* to *C1* with respect to *GOAL* is handled. When we check the table, there are two linguistic terms, *HI* and *EI*, and the term “-” which means null. The intuitionistic fuzzy numbers that correspond to the *HI* and *EI* terms will be used for the calculations. We can consider *HI* an optimistic value and *EI* a pessimistic value for this comparison. All comparisons were made according to this logic in Table 2.

For the next step, linguistic terms were transformed to IFNs. Table 3 presents the collected pairwise comparison matrices which correspond with the IFNs.

Table 3
Collected pairwise comparison matrices that correspond with the IFNs

Goal	**C1**	**C2**	**C3**	**C4**
C1	<[0.5, 0.5]>, <[0.5, 0.5]>, <[0.5, 0.5]>	<[0.35, 0.55]>, <[0.5, 0.5]>	<[0.5, 0.5]>, <[0.35, 0.55]>	<[0.55, 0.35]>
C2	<[0.55, 0.35]>, <[0.5, 0.5]>	<[0.5, 0.5]>, <[0.5, 0.5]>, <[0.5, 0.5]>	<[0.35, 0.55]>, <[0.5, 0.5]>, <[0.2, 0.7]>	<[0.7, 0.2]>, <[0.55, 0.35]>
C3	<[0.5, 0.5]>, <[0.55, 0.35]>	<[0.55, 0.35]>, <[0.5, 0.5]>, <[0.7, 0.2]>	<[0.5, 0.5]>, <[0.5, 0.5]>, <[0.5, 0.5]>	<[0.7, 0.2]>, <[0.85, 0.05]>
C4	<[0.35, 0.55]>	<[0.2, 0.7]>, <[0.35, 0.55]>	<[0.2, 0.7]>, <[0.05, 0.85]>	<[0.5, 0.5]>, <[0.5, 0.5]>, <[0.5, 0.5]>
C1	**AL1**	**AL2**	**AL3**	**AL4**
AL1	<[0.5, 0.5]>, <[0.5, 0.5]>, <[0.5, 0.5]>	<[0.35, 0.55]>, <[0.5, 0.5]>	<[0.5, 0.5]>, <[0.35, 0.55]>	<[0.55, 0.35]>
AL2	<[0.55, 0.35]>, <[0.5, 0.5]>	<[0.5, 0.5]>, <[0.5, 0.5]>, <[0.5, 0.5]>	<[0.35, 0.55]>, <[0.5, 0.5]>, <[0.2, 0.7]>	<[0.7, 0.2]>, <[0.55, 0.35]>
AL3	<[0.5, 0.5]>, <[0.35, 0.55]>	<[0.55, 0.35]>, <[0.5, 0.5]>, <[0.7, 0.2]>	<[0.5, 0.5]>, <[0.5, 0.5]>, <[0.5, 0.5]>	<[0.7, 0.2]>, <[0.85, 0.05]>
AL4	<[0.35, 0.55]>	<[0.2, 0.7]>, <[0.35, 0.55]>	<[0.2, 0.7]>, <[0.05, 0.85]>	<[0.5, 0.5]>, <[0.5, 0.5]>, <[0.5, 0.5]>
C2	**AL1**	**AL2**	**AL3**	**AL4**
AL1	<[0.5, 0.5]>, <[0.5, 0.5]>, <[0.5, 0.5]>	<[0.35, 0.55]>, <[0.55, 0.35]>	<[0.55, 0.35]>	<[0.5, 0.5]>, <[0.55, 0.35]>
AL2	<[0.55, 0.35]>, <[0.35, 0.55]>	<[0.5, 0.5]>, <[0.5, 0.5]>, <[0.5, 0.5]>	<[0.2, 0.7]>, <[0.7, 0.2]>	<[0.55, 0.35]>, <[0.5, 0.5]>, <[0.35, 0.55]>
AL3	<[0.35, 0.55]>	<[0.7, 0.2]>, <[0.2, 0.7]>	<[0.5, 0.5]>, <[0.5, 0.5]>, <[0.5, 0.5]>	<[0.2, 0.7]>, <[0.35, 0.55]>
AL4	<[0.5, 0.5]>, <[0.35, 0.55]>	<[0.35, 0.55]>, <[0.5, 0.5]>, <[0.55, 0.35]>	<[0.7, 0.2]>, <[0.55, 0.35]>	<[0.5, 0.5]>, <[0.5, 0.5]>, <[0.5, 0.5]>
C3	**AL1**	**AL2**	**AL3**	**AL4**
AL1	<[0.5, 0.5]>, <[0.5, 0.5]>, <[0.5, 0.5]>	<[0.35, 0.55]>, <[0.55, 0.35]>, <[0.7, 0.2]>	<[0.55, 0.35]>, <[0.35, 0.55]>	<[0.5, 0.5]>, <[0.55, 0.35]>
AL2	<[0.55, 0.35]>, <[0.35, 0.55]>, <[0.2, 0.7]>	<[0.5, 0.5]>, <[0.5, 0.5]>, <[0.5, 0.5]>	<[0.2, 0.7]>, <[0.55, 0.35]>	<[0.55, 0.35]>, <[0.5, 0.5]>, <[0.35, 0.55]>
AL3	<[0.35, 0.55]>, <[0.55, 0.35]>	<[0.7, 0.2]>, <[0.35, 0.55]>	<[0.5, 0.5]>, <[0.5, 0.5]>, <[0.5, 0.5]>	<[0.2, 0.7]>, <[0.35, 0.55]>
AL4	<[0.5, 0.5]>, <[0.35, 0.55]>	<[0.35, 0.55]>, <[0.5, 0.5]>, <[0.55, 0.35]>	<[0.7, 0.2]>, <[0.55, 0.35]>	<[0.5, 0.5]>, <[0.5, 0.5]>, <[0.5, 0.5]>
C4	**AL1**	**AL2**	**AL3**	**AL4**
AL1	<[0.5, 0.5]>, <[0.5, 0.5]>, <[0.5, 0.5]>	<[0.35, 0.55]>, <[0.55, 0.35]>	<[0.55, 0.35]>, <[0.5, 0.5]>	<[0.5, 0.5]>, <[0.55, 0.35]>, <[0.55, 0.35]>
AL2	<[0.55, 0.35]>, <[0.35, 0.55]>	<[0.5, 0.5]>, <[0.5, 0.5]>, <[0.5, 0.5]>	<[0.2, 0.7]>, <[0.35, 0.55]>, <[0.55, 0.35]>	<[0.55, 0.35]>, <[0.5, 0.5]>, <[0.35, 0.55]>
AL3	<[0.35, 0.55]>, <[0.5, 0.5]>	<[0.7, 0.2]>, <[0.55, 0.35]>, <[0.35, 0.55]>	<[0.5, 0.5]>, <[0.5, 0.5]>, <[0.5, 0.5]>	<[0.55, 0.35]>, <[0.35, 0.55]>, <[0.35, 0.55]>
AL4	<[0.5, 0.5]>, <[0.35, 0.55]>, <[0.35, 0.55]>	<[0.35, 0.55]>, <[0.5, 0.5]>, <[0.55, 0.35]>	<[0.35, 0.55]>, <[0.55, 0.35]>, <[0.55, 0.35]>	<[0.5, 0.5]>, <[0.5, 0.5]>, <[0.5, 0.5]>

For the next step, the minimum and maximum values of the membership and non-membership values are obtained as shown in Table 4.

Table 4
Minimum and maximum vales of the membership and non-membership values

	μ_{min}	μ_{max}	ϑ_{min}	ϑ_{max}	μ_{min}	μ_{max}	ϑ_{min}	ϑ_{max}	μ_{min}	μ_{max}	ϑ_{min}	ϑ_{max}	μ_{min}	μ_{max}	ϑ_{min}	ϑ_{max}
Goal	**C1**				**C2**				**C3**				**C4**			
C1	0.5	0.5	0.5	0.5	0.35	0.55	0.5	0.55	0.35	0.5	0.5	0.55	0.55	0.55	0.35	0.35
C2	0.5	0.55	0.35	0.5	0.5	0.5	0.5	0.5	0.2	0.5	0.5	0.7	0.55	0.7	0.2	0.35
C3	0.5	0.55	0.35	0.5	0.5	0.5	0.2	0.5	0.5	0.5	0.5	0.5	0.7	0.85	0.05	0.2
C4	0.35	0.35	0.55	0.55	0.2	0.7	0.55	0.7	0.05	0.2	0.7	0.85	0.5	0.5	0.5	0.5
C1	**AL1**				**AL2**				**AL3**				**AL4**			
AL1	0.5	0.5	0.5	0.5	0.35	0.55	0.5	0.55	0.35	0.5	0.5	0.55	0.55	0.55	0.35	0.35
AL2	0.5	0.55	0.35	0.5	0.5	0.5	0.5	0.5	0.2	0.5	0.5	0.7	0.55	0.7	0.2	0.35
AL3	0.35	0.5	0.5	0.55	0.5	0.5	0.2	0.5	0.5	0.5	0.5	0.5	0.7	0.85	0.05	0.2
AL4	0.35	0.35	0.55	0.55	0.2	0.7	0.55	0.7	0.05	0.2	0.7	0.85	0.5	0.5	0.5	0.5
C2	**AL1**				**AL2**				**AL3**				**AL4**			
AL1	0.5	0.5	0.5	0.5	0.35	0.55	0.35	0.55	0.55	0.55	0.35	0.35	0.5	0.55	0.35	0.5
AL2	0.35	0.55	0.35	0.55	0.5	0.5	0.5	0.5	0.2	0.7	0.2	0.7	0.35	0.55	0.35	0.55
AL3	0.35	0.35	0.55	0.55	0.2	0.7	0.2	0.7	0.5	0.5	0.5	0.5	0.2	0.35	0.55	0.7
AL4	0.35	0.5	0.5	0.55	0.35	0.55	0.35	0.55	0.55	0.7	0.2	0.35	0.5	0.5	0.5	0.5
C3	**AL1**				**AL2**				**AL3**				**AL4**			
AL1	0.5	0.5	0.5	0.5	0.35	0.55	0.2	0.55	0.35	0.55	0.35	0.55	0.5	0.55	0.35	0.5
AL2	0.2	0.55	0.35	0.7	0.5	0.5	0.5	0.5	0.2	0.55	0.35	0.7	0.35	0.55	0.35	0.55
AL3	0.35	0.55	0.35	0.55	0.35	0.55	0.2	0.55	0.5	0.5	0.5	0.5	0.2	0.35	0.55	0.7
AL4	0.35	0.5	0.5	0.55	0.35	0.55	0.35	0.55	0.55	0.7	0.2	0.35	0.5	0.5	0.5	0.5
C4	**AL1**				**AL2**				**AL3**				**AL4**			
AL1	0.5	0.5	0.5	0.5	0.35	0.55	0.35	0.55	0.5	0.55	0.35	0.5	0.5	0.55	0.35	0.5
AL2	0.35	0.55	0.35	0.55	0.5	0.5	0.5	0.5	0.2	0.55	0.35	0.7	0.35	0.55	0.35	0.55
AL3	0.35	0.5	0.5	0.55	0.35	0.55	0.2	0.55	0.5	0.5	0.5	0.5	0.35	0.55	0.35	0.55
AL4	0.35	0.5	0.5	0.55	0.35	0.55	0.35	0.55	0.35	0.55	0.35	0.55	0.5	0.5	0.5	0.5

ISSN 1936-6744
https://doi.org/10.13033/ijahp.v11i1.610

Then, the values in Table 4 were used to obtain optimistic and pessimistic weights of the criteria and alternatives in the comparison matrices. To obtain an optimistic value of a comparison, the maximum of the membership and minimum of the non-membership values was taken. Similarly, to obtain a pessimistic value of a comparison, the minimum of the membership and maximum of the non-membership values was taken.

To clarify this step, an example is given based on the comparison of all criteria with *C1* with respect to *GOAL* as shown in Table 5.

Table 5
Optimistic results based on the comparison of all criteria with C1 with respect to GOAL

Comparison		μ	ϑ	$\mathfrak{H}$
C1	C1	0.50	0.50	0.50
C2	C1	0.55	0.35	0.60
C3	C1	0.55	0.35	0.60
C4	C1	0.35	0.55	0.40

Here, when we examined the values in Table 4, μ and ϑ values for the optimistic results were calculated as below, respectively:

For *C1*-*C1*, μ value $\max_{\mu\, C1-C1}(0.5{,}0.5) = 0.5$, and ϑ value $\min_{\vartheta\, C1-C1}(0.5{,}0.5) = 0.5$.

For *C2*-*C1*, μ value $\max_{\mu\, C2-C1}(0.50{,}0.55) = 0.55$, and ϑ value $\min_{\vartheta\, C2-C1}(0.35{,}0.5) = 0.35$. The same calculations were performed for every comparison.

After finding the membership and non-membership functions, Equation 9 was applied to find the defuzzified values.

Similar logic was applied for the pessimistic results. This time, the minimum of μ values and maximum of ϑ values were selected. The results are given in Table 6.

Table 6
Pessimistic results based on the comparison of all criteria with C1 with respect to GOAL

Comparison		μ	ϑ	$\mathfrak{H}$
C1	C1	0.50	0.50	0.50
C2	C1	0.50	0.50	0.50
C3	C1	0.50	0.50	0.50
C4	C1	0.35	0.55	0.40

These values were used to obtain the unnormalized weights. For the comparison of criteria with respect to GOAL, the optimistic and pessimistic values of *C1* were found as shown below, respectively:

ISSN 1936-6744
https://doi.org/10.13033/ijahp.v11i1.610

Optimistic value= $\frac{0.5+0.525+0.5+0.6}{4} = 0.531$
Pessimistic value= $\frac{0.5+0.4+0.4+0.6}{4} = 0.475$

The results of these calculations are given in Table 7.

Table 7
Unnormalized weights of the criteria and alternatives

GOAL	**Optimistic**	**Pessimistic**
C1	0.531	0.475
C2	0.588	0.463
C3	0.663	0.563
C4	0.431	0.313
C1	**Optimistic**	**Pessimistic**
AL1	0.531	0.475
AL2	0.588	0.463
AL3	0.638	0.538
AL4	0.431	0.313
C2	**Optimistic**	**Pessimistic**
AL1	0.575	0.5
AL2	0.613	0.388
AL3	0.513	0.35
AL4	0.588	0.475
C3	**Optimistic**	**Pessimistic**
AL1	0.594	0.45
AL2	0.575	0.35
AL3	0.544	0.388
AL4	0.588	0.475
C4	**Optimistic**	**Pessimistic**
AL1	0.575	0.475
AL2	0.575	0.388
AL3	0.569	0.425
AL4	0.55	0.425

Finally, mid values and local weights of alternatives and weights of criteria are presented in Table 8.

Table 8
Mid values and local weights of alternatives and weights of criteria

GOAL	Mid-Point	Normalized Weight
C1	0.503	0.25
C2	0.525	0.261
C3	0.613	0.304
C4	0.372	0.185
C1	**Mid-Point**	**Normalized Weight**
AL1	0.503	0.253
AL2	0.525	0.264
AL3	0.588	0.296
AL4	0.372	0.187
C2	**Mid-Point**	**Normalized Weight**
AL1	0.538	0.269
AL2	0.5	0.25
AL3	0.431	0.216
AL4	0.531	0.266
C3	**Mid-Point**	**Normalized Weight**
AL1	0.522	0.263
AL2	0.463	0.233
AL3	0.466	0.235
AL4	0.531	0.268
C4	**Mid-Point**	**Normalized Weight**
AL1	0.525	0.264
AL2	0.481	0.242
AL3	0.497	0.25
AL4	0.488	0.245

Global weights of the alternatives were calculated as follows:

AL1=0.262
AL2=0.247
AL3=0.248
AL4=0.243

To clarify the calculations, *AL1*'s normalized weight was obtained by using the values given in Table 7 as below.

$$0.25 * 0.253 + 0.261 * 0.268 + 0.304 * 0.263 + 0.185 * 0.264 = 0.262$$

Similarly, the same calculation was also performed for the other alternatives and the global weights were found. After the calculations, *AL1* was the best alternative for investment. The worst alternative was determined to be *AL4*.

3.2 Sensitivity analysis

To determine the robustness of the proposed approach, we conducted a one-at-a-time sensitivity analysis with respect to changes of criteria weights and then observed the alternatives ranks. Table 9 represents the pattern of the analysis and results.

Table 9
Pattern for the sensitivity analysis and rank of the alternatives

	AL1	AL2	AL3	AL4
C1 – CLI	1	3	4	2
C1 – EI	1	3	2	4
C1 – CHI	2	3	1	4
C2 – CLI	1	3	2	4
C2 – EI	1	2	4	3
C2 – CHI	1	3	4	2
C3 – CLI	1	3	2	4
C3 – EI	1	4	3	2
C3 – CHI	1	4	3	2
C4 – CLI	1	3	4	2
C4 – EI	1	3	2	4
C4 – CHI	1	3	2	4

To illustrate with an example, C1's weight was changed as CLI which corresponds to $[0.05, 0.85]$. Then, this intuitionistic fuzzy number was converted to a crisp value by using the defuzzification function. Next, new results were obtained by using this new weight. Finally, the normalized weights of alternatives were found. Figure 1 presents the ranks of alternatives based on changes in criteria weights.

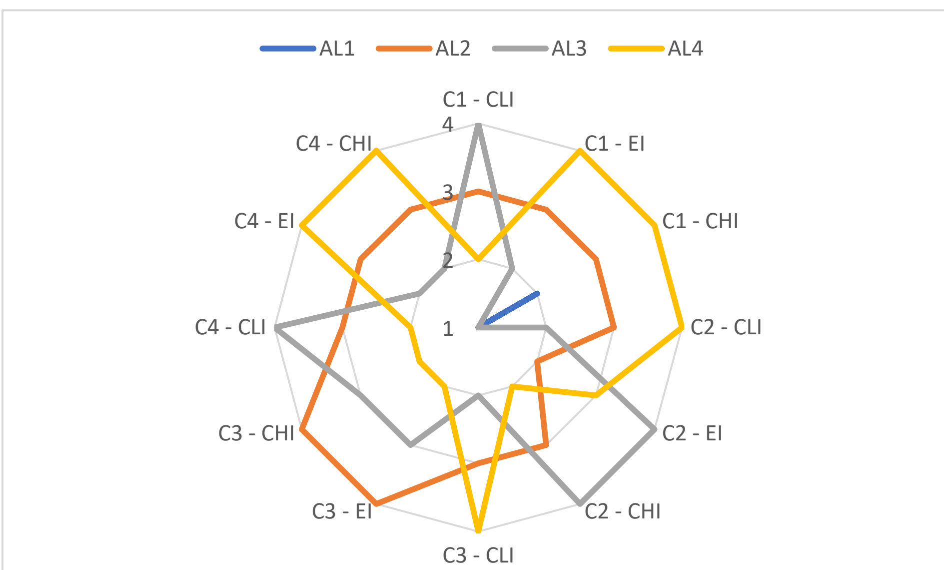

Figure 1 Results of the sensitivity analyses

Through the calculations, it can be seen that *AL1* is always the best alternative except for one criterion weight change (when C1 is equal to CHI). This verifies that our proposed

ISSN 1936-6744
https://doi.org/10.13033/ijahp.v11i1.610

method's decisions are robust. The weights also changed in every change of criteria weight. This also verifies that our model is sensitive to the changes in the criteria values.

4. Conclusion

In this paper, a hesitant intuitionistic AHP method was proposed and applied to an illustrative example to show its applicability. Also, a one-at-a-time sensitivity analysis was conducted to show that the proposed method's decisions are robust. Since intuitionistic fuzzy sets offer a larger domain to represent the decision makers' judgments, it is very useful for the application. Hesitant fuzzy sets help decision makers assign different values to the same element.

The result of the proposed method shows that the *C3* criterion is the most effective criterion for the selection process. As a result of the application, *AL1* is selected as the best alternative. Through the application process, a one-at-a-time sensitivity analysis was applied, and it was revealed that the results of the proposed method were robust. Thus, our proposed methodology was an evaluation process which can be used as a decision-making tool by managers or researchers to make useful inferences, judgements, and decisions for real-world applications. Since the model does not only consider the quantitative data, but also qualitative data, it is very useful for areas that have uncertainty and indeterminacy in the decision making processes.

For further research, the data can be extended by using a real case problem's input for business, energy, and facility location planning problems. Also, an integrated decision making process consists of our proposed method and a fuzzy inference system can be used and the obtained results can be compared with other decision making methods such as TOPSIS, VIKOR, and ELECTRE.

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