SIMPLE AND OBJECTIVE DETERMINATION OF CRITERIA WEIGHTS FOR EVALUATING ALTERNATIVES WHEN USING THE ANALYTIC HIERARCHY PROCESS

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ABSTRACT

In both design and project management, it is usually necessary to make decisions based on criteria that, in many cases, are not completely objective. One of the most widely used methods to assist in multi-criteria decision making is the Analytic Hierarchy Process (AHP). The construction of the pairwise comparison matrix is the most critical element of the AHP method. Its construction usually requires a group of experts to make value judgments among the criteria for evaluating alternatives. This article presents a simple and objective method to construct the AHP pairwise comparison matrix, thereby reducing the degree of subjectivity. This contribution focuses on the step of assigning the scale values (1-9) to build the criteria matrix. The only step that requires human intervention is the ranking of the criteria. The rest of the steps can be programmed, e.g., in a spreadsheet, and performed automatically. Two options are presented to relate the criteria based on their positions in the ranking: option A by means of the ratios of the order of the criteria, and option B by means of the differences in the order of the criteria. The applicability of both options is evaluated using hypothetical cases compared to the rank-order centroid weight method, and an example taken from the literature. Both options of the method give good results, but option B seems to be more appropriate for problems with more than five criteria. Option A would be the most appropriate for problems with three to five criteria.

Keywords: AHP; criteria weights; pairwise comparison; MDCA; project management

1. Introduction

In both design and project management, it is usually necessary to make decisions based on criteria that, in many cases, are not completely objective. Subjectivity in decision making may appear, especially at three moments in the process: a) when choosing the criteria to be used for decision making, b) when determining the relative weights of these criteria, and c) in some cases, when evaluating the alternatives (when there is no deterministic tool available to compute the alternative's performance level). A large number of methods have been published in the literature to assist in multicriteria decision making, such as those collected in several review articles (Alvarez et al., 2021; Cinelli et al., 2020, 2022; Kou et al., 2020). Among these methods, the Analytic Hierarchy Process (AHP) stands out as one of the most widely used due to its age and applicability.

The AHP is a powerful decision-making technique developed by mathematician and management scientist Thomas L. Saaty in the 1970s (Saaty, 1980). It has since gained widespread acceptance in various fields due to its ability to address complex problems in a systematic and structured manner. The AHP proves particularly useful when facing decisions involving multiple criteria and alternatives, which can otherwise complicate the decision-making process. Its applications span across diverse disciplines such as project management (Ahmad & Laplante, 2006; Al-Harbi, 2001; Gunduz & Alfar, 2019) strategic planning (Ardil, 2021a; Djamel et al., 2021; Sisto et al., 2022), equipment selection (Ardil, 2021a; Bodziony et al., 2019; Santos et al., 2021), material selection (A. Kumar & Kumar, 2019; R. Kumar et al., 2021), supplier selection (Chen, 2021; Tiganis et al., 2023), risk evaluation (Andreolli et al., 2022), construction (Zhu et al., 2021), education (Ardil, 2021b) and more.

According to the taxonomy proposed by Cinelli et al. (2022), the AHP is classified as a preference model within the group of scoring functions. It includes a normalization procedure to apply a form of aggregation to derive an overall score that can be used to rank, sort, or choose the most preferred alternative. The AHP determines the priorities of the alternatives by pairwise comparisons of the alternatives provided by the decision maker (DM) with respect to each criterion. The intensity of preference of these pairwise comparisons is expressed on a ratio scale (typically between 1 and 9).

The fundamental idea behind the AHP is to decompose a complex decision problem into a hierarchical structure of criteria and sub-criteria, which facilitates a comparative analysis of different factors and their relative importance. By assigning numerical values to these elements through pairwise comparisons, the method quantifies subjective judgments, transforming qualitative inputs into a robust quantitative framework (Forman & Gass, 2001; Karpak, 2022; Saaty, 2008; Siekelova et al., 2021; Vargas, 1990).

It is worth highlighting that, from the seminal work of Saaty (1980), several versions of the AHP have been introduced, through minor modifications of the ranking process and the resulting computation of criteria's weights. For instance, in Sangiorgio et al. (2021), the authors propose using a mechanism inspired from the Simos-Roy-Figuera method for ELECTRE (Figueira & Roy, 2002) instead of the matrix of pairwise comparisons among criteria. Concretely, criteria comparison is formalized as a card game, where each criterion has its associated card, and a ranking is determined in two phases: first, a preliminary ordering from the least to the most important criterion is established by setting a specific order of their corresponding cards; then, blank cards are added in between two consecutive cards (i.e., criteria) in order to represent the magnitude of the difference between criteria (for a major difference of importance between two criteria, more blank cards will be inserted between the two cards of the corresponding criteria). Note that two criteria with similar importance may be assigned the same rank. Finally, from the obtained ranking, criteria's weights are simply computed as the normalized rank values. A similar procedure is applied to obtain decision-makers' preferences regarding alternatives, in order to compute the final priorities through the classical Saaty's formula (Saaty, 1980). This technique

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was subsequently enhanced for the area of construction, by integrating an Augmented Reality tool that provides decision-makers with a 3D representation of the different building designs, in order to aid the decision-making process.

While the Analytic Hierarchy Process (AHP) is a valuable decision-making tool, it is not without its challenges and difficulties. Some of the most important difficulties in applying AHP are detailed in Asadabadi et al. (2019), Bana e Costa and Vansnick (2008), Bruno et al. (2011), Holder (1990), Munier and Hontoria (2021) and Taylor et al. (1998):

1. Subjectivity and bias: The AHP relies heavily on pairwise comparisons to derive relative importance between criteria and also between alternatives. However, these comparisons are often subjective and can be influenced by individual biases, leading to potential inaccuracies in the final results.
2. Inconsistent judgments: DMs may provide inconsistent judgments during the pairwise comparisons, leading to inconsistencies in the AHP model which affects the overall reliability of the results.
3. Large and complex hierarchies: Handling extensive hierarchies with numerous criteria and sub-criteria can become cumbersome and time-consuming. As the hierarchy grows, the complexity of the model increases, making it challenging to manage and analyze.
4. Data collection and validation: Gathering accurate and relevant data for pairwise comparisons can be difficult, especially when dealing with diverse and heterogeneous criteria. Additionally, validating the data and ensuring its reliability can be a time-consuming task.
5. Sensitivity to changes: AHP results can be sensitive to even slight changes in the input data or judgments. Small adjustments in the comparisons could potentially lead to significant variations in the final outcomes, making it crucial to validate and verify the robustness of the results.
6. Lack of consensus: In group decision-making scenarios, achieving consensus on pairwise comparisons can be challenging. Conflicting opinions among DMs may hinder the process and delay reaching a final decision.
7. Weighting issues: Determining the appropriate weighting for criteria and sub-criteria is critical in the AHP. However, the method for calculating these weights can be subjective and open to interpretation, leading to potential bias in the decision-making process.
8. Resource-intensive: Implementing the AHP can require significant time and effort, especially when dealing with complex decision problems and large datasets. It requires expertise in structuring the hierarchy, conducting pairwise comparisons, and interpreting the results.
9. Lack of transparency: AHP results can be difficult to interpret and explain to stakeholders who are not familiar with the methodology. This lack of transparency may lead to skepticism and resistance towards accepting the final decisions.

Despite these challenges, the Analytic Hierarchy Process remains a valuable decision-making technique when applied thoughtfully and with awareness of its limitations. By recognizing and addressing these difficulties, DMs can enhance the effectiveness and reliability of the AHP application in solving real-world problems. The above-mentioned issues 1, 3, 4, 5 and 8 are mostly related to the characteristics of the problem to solve or the practical implementation of the AHP method (point 8). However, aspects that are related to the method itself can be found in the issues 1, 2, 6 and 7.

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Perhaps one of the critical points where the greatest uncertainty in the results of the application of the AHP method is introduced is the assignment of a value on the 1-9 scale to a criterion during the pairwise comparison between criteria. Using ranks to derive weights through some formulas seems more reliable than simply assigning weights directly to criteria (or values on the 1-9 scale), because DMs are usually more confident about the ranks of some criteria than about their weights, and can agree on ranks more easily (Roszkowska, 2013). A number of methods for assigning weights to the criteria using formulas based on an order of priority have been published in the literature (Hutton Baron, 1992; Lootsma, 1996; Solvmosi & Dombi, 1986; Stillwell et al., 1981). Among these methods, the rank-order centroid weight (ROCW) method stands out as superior in terms of accuracy and ease of use (Barron & Barrett, 1996; Edwards & Barron, 1994; Noh & Lee, 2003; Olson & Dorai, 1992; Roskowska, 2013).

In this article, we propose a method to make the application of the AHP method more objective, by introducing a computational method for assigning values on the 1-9 scale in the criteria pairwise comparison step, starting from a defined order of the criteria. Thus, the proposed method addresses the above-mentioned issues 1, 2, 6 and 7 in the application of the AHP.

It is worth mentioning that the proposed method is not intended to change the other steps of the AHP method. The objective is to facilitate the process of assigning values during the pairwise comparison of criteria. Although the method starts from an ordered list of criteria, the weights of these criteria are calculated according to the AHP method, unlike the above-mentioned methods that assign the weights, such as ROCW. Therefore, in this case, the calculation of the criteria weights and the consistency check of the criteria matrix are similar to those of the original AHP method.

2. Proposed methodology

Pairwise comparison is a major part of the AHP. It is used to weight the identified criteria. The first step of determining the weights includes the creation of a judgement matrix based on the judgement about the relative importance of each criterion according to a linear scale from 1 to 9 (Figure 1). This scale serves to define how many times one criterion is better than another and how many times one criterion is preferred over another one (Ishizaka & Labib, 2011).

Thus, if we consider n criteria C_x, a square matrix n × n of comparisons between these criteria can be built:

$$M = \begin{bmatrix} v_{11} & v_{12} & \cdots & v_{1n} \\ v_{21} & v_{22} & \cdots & v_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ v_{n1} & v_{n2} & \cdots & v_{nn} \end{bmatrix} = \begin{bmatrix} 1 & v_{12} & \cdots & v_{1n} \\ 1/v_{12} & 1 & \cdots & v_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ 1/v_{1n} & 1/v_{2n} & \cdots & 1 \end{bmatrix} \quad (1)$$

Where v_{ij} represents the importance value of criterion C_i with respect to criterion C_j, according to the scale 1-9. Therefore, all the elements v_{ii} are equal to 1 in the diagonal of the matrix M, which stands for equal importance between the criterion C_i and itself. But, below (or above) the diagonal, the elements v_{ij} are equal to the inverse of their value ($1/v_{ij}$) in the lower (or upper) half of M.

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In the second step of the AHP method, the criteria weights are calculated by transforming the matrix *M* (Saaty, 1980; 2008). Finally, in the third step a consistency check is done to see if the comparison represented by the matrix *M* is reasonable (Saaty, 1980; 2008).

The modification we propose below only refers to the first step of the original AHP method as formulated by Saaty (1980).

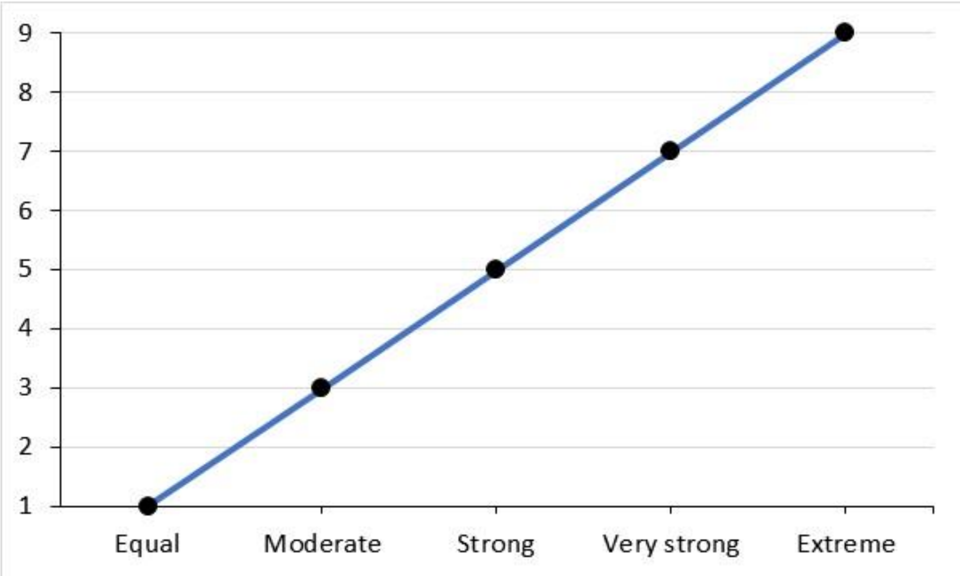

Figure 1 Scale of preference for the comparison of two elements (criteria or alternatives) (adapted from Saaty (1980))

Assigning a value of 1-9 to the intensity of importance by comparing two different criteria is not obvious in many cases. Nevertheless, it is a proven fact that people find it easier to rank a list of criteria from most important to least important than to assign a specific value to them (Bailey, 1982; Munier & Hontoria, 2021). In this sense, we propose following the procedure outlined in Figure 2 for the construction of the matrix *M*.

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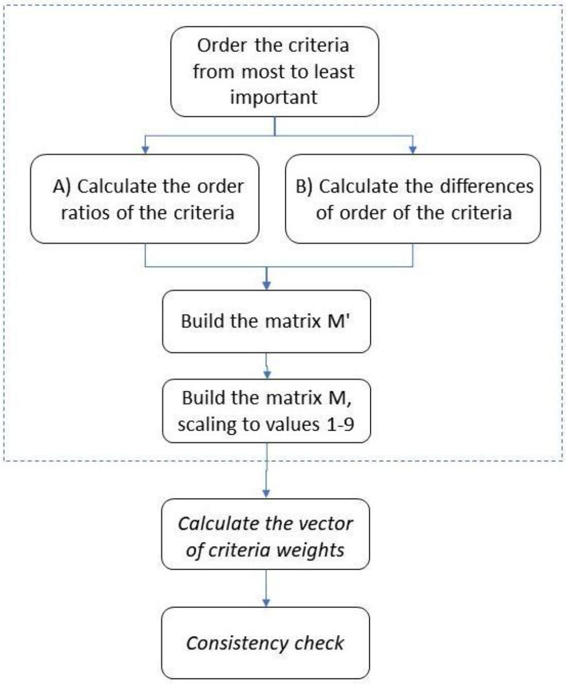

Figure 2 Procedure to determine the weights of the criteria according to the AHP method (steps indicated in the square with dashed lines corresponding to the modification proposed in this work)

Suppose there are n criteria (C_1, C_2, …, C_n). These are the steps that should be followed to determine their weights according to the procedure recommended here (the proposed modification affects the first four steps):

1) Sort the n criteria in order of importance; there may be criteria with equal importance. Therefore, the list of order numbers will go from 1 to p ($p \leq n$), where 1 corresponds to the criterion of highest importance (C_{max}) and p to the criterion of lowest importance (C_{min}).

2) Relate the order numbers (c) of the criteria (C) in the above list. This can be done according to the following two options:

   A) By the ratio c_i/c_j of the order numbers of the criteria C_i and C_j, respectively, with i and j = 1, 2, ..., p. This ratio will have a range of values between $1/p$ and p. The lower the ratio is, the greater the importance of C_i with respect to C_j.
   B) By the difference $c_i - c_j$ of the order numbers of the criteria C_i and C_j, respectively, with i and j = 1, 2, ..., p. This difference will have a range of values between $1–p$ and $p–1$. The more negative the difference is, the greater the importance of C_i with respect to C_j.

3) Construct the square matrix M' n x n resulting from the comparison of the criteria according to option A or B of the previous step.

4) Obtain the matrix M, transforming the coefficients of the matrix M' into values of the scale 1-9, by linear interporlation depending on option A or B:

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A) If $c_i/c_j < 1$, then $v_{ij} = \frac{\left(\frac{c_j}{c_i}-1\right)*8}{p-1} + 1$
If $c_i/c_j = 1$, then $v_{ij} = 1$ (2)
If $c_i/c_j > 1$, then $v_{ij} = \frac{1}{\frac{\left(\frac{c_i}{c_j}-1\right)*8}{p-1}+1}$

B) If $c_i - c_j < 0$, then $v_{ij} = \frac{8}{p-1} * \left|c_i - c_j\right| + 1$
If $c_i - c_j = 0$, then $v_{ij} = 1$ (3)
If $c_i - c_j > 0$, then $v_{ij} = \frac{1}{\frac{8}{p-1}*\left|c_i-c_j\right|+1}$

5) Calculate the vector of criteria weights or priority vector or eigenvector (*W*), following the AHP method:
   - Sum each column *j* of the matrix *M*.
   - Normalize the values v_{ij} of *M* by dividing the sum of their column *j* to obtain the matrix *M**
   - The weight w_i of C_i is obtained by averaging the values v_{ij} of the row *i* of *M**, such that $\sum_{i=1}^{n} w_i = 1$ where $w_i \geq 0$, $i = 1, 2, \ldots, n$.

6) Perform a consistency test like the one described in the formulation of the AHP method (Saaty, 1980). To check the matrix *M* for consistency, first the consistency vector (*K*) is obtained by multiplying the not standardized pairwise comparison matrix (*M*) with the priority vector (*W*), and then dividing by *W*:

$$[K] = \frac{[M]\times[W]}{[W]} \tag{4}$$

Out of the values of K, their mean value is calculated. This mean value represents the average consistency of the matrix and is also called the eigenvalue (λ):

$$\lambda = \frac{[W]}{n} \tag{5}$$

The degree of inconsistency of *M* is evaluated by the extent to which the highest eigenvalue (λ_{max}) of the vector *K* deviates from the order of the matrix (*n*) (Saaty, 1980). Then, the consistency index (i_c) is calculated as follows:

$$i_c = \frac{\lambda_{max} - n}{n-1} \tag{6}$$

If *M* were perfectly consistent, i_c should be 0. This index is then divided by a random consistency index (i_r) to obtain the consistency ratio (r_c). The i_r depends on the number of criteria; for instance, it is 0.52 for $n = 3$, and 1.58 for $n = 15$ (Alonso & Lamata, 2006; Cho, 2019; Franek & Kresta, 2014). If r_c is less than 0.1 (for $n \geq 3$), this means that the matrix *M* is consistent; otherwise, it is inconsistent:

$$r_c = \frac{i_c}{i_r} \quad (< 0.1 \text{ for consistency}) \tag{7}$$

This procedure was applied to a number of hypothetical cases. The priority vectors obtained by applying options A and B in step 2 were compared with the vector of

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weights obtained by the ROCW method. According to this approach, the expected value of the weights can be calculated using the following formula (Barron & Barrett, 1996; Roskowska, 2013):

$$w_{ei} = \frac{1}{n} \sum_{j=i}^{n} \frac{1}{c_j} \tag{8}$$

Where w_{ei} are the criteria weights estimated from the order numbers c_j of the criteria.

Finally, both options were applied to an example taken from the literature and the results were compared.

Taking as reference the vector of weights according to the ROCW method in the hypothetical cases and the vector of priorities of the literature example, the deviations (absolute errors, *AE*) of the weights obtained by options A and B were calculated. Then, the mean relative error (*MRE*) and the mean square error (*RMSE*) were also calculated as follows:

$$MRE = \frac{1}{n} \sum_{i=1}^{n} \frac{AE_i}{w_{ei}} = \frac{1}{n} \sum_{i=1}^{n} \frac{(w_i - w_{ei})}{w_{ei}} \tag{9}$$

$$RMSE = \frac{1}{n} \sum_{i=1}^{n} (w_i - w_{ei})^2 \tag{10}$$

Where w_{ei} are the criteria weights estimated from the reference.

3. Numerical experiments and results

3.1. Hypothetical cases

We applied this procedure to 13 hypothetical cases with three to 15 criteria, in which all criteria are assumed to have unequal importance. Therefore, $p = n$. The cases are described in Table 1.

Table 1

Hypothetical cases analyzed

Case no.	Number of criteria	Order of importance
1	3	$C_1 > C_2 > C_3$
2	4	$C_1 > C_2 > C_3 > C_4$
3	5	$C_1 > C_2 > C_3 > C_4 > C_5$
4	6	$C_1 > C_2 > C_3 > C_4 > C_5 > C_6$
5	7	$C_1 > C_2 > C_3 > C_4 > C_5 > C_6 > C_7$
6	8	$C_1 > C_2 > C_3 > C_4 > C_5 > C_6 > C_7 > C_8$
7	9	$C_1 > C_2 > C_3 > C_4 > C_5 > C_6 > C_7 > C_8 > C_9$
8	10	$C_1 > C_2 > C_3 > C_4 > C_5 > C_6 > C_7 > C_8 > C_9 > C_{10}$
9	11	$C_1 > C_2 > C_3 > C_4 > C_5 > C_6 > C_7 > C_8 > C_9 > C_{10} > C_{11}$
10	12	$C_1 > C_2 > C_3 > C_4 > C_5 > C_6 > C_7 > C_8 > C_9 > C_{10} > C_{11} > C_{12}$
11	13	$C_1 > C_2 > C_3 > C_4 > C_5 > C_6 > C_7 > C_8 > C_9 > C_{10} > C_{11} > C_{12} > C_{13}$
12	14	$C_1 > C_2 > C_3 > C_4 > C_5 > C_6 > C_7 > C_8 > C_9 > C_{10} > C_{11} > C_{12} > C_{13} > C_{14}$
13	15	$C_1 > C_2 > C_3 > C_4 > C_5 > C_6 > C_7 > C_8 > C_9 > C_{10} > C_{11} > C_{12} > C_{13} > C_{14} > C_{15}$

For illustrative purposes, the following is a step-by-step application of the proposed method to the hypothetical case #1 (three criteria). In this case, criteria C_1, C_2 and C_3 are prioritized as shown in Table 2.

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Table 2
Order of the criteria for the hypothetical case #1

Criteria	Order
C_1	1
C_2	2
C_3	3

Table 3 shows the matrices *M'*, *M*, and *M**, respectively, which are a result of applying the proposed method to case #1 (three criteria), for both options A and B. The priority vectors for both options A and B are also shown in Table 3. These results refer to steps 2 to 5 of the above method.

Table 3
Application of the proposed method to case #1 (three criteria) for options A and B

	Option A				Option B			
	Matrix *M'*							
	C_1	C_2	C_3		C_1	C_2	C_3	
C_1	1	2	3		0	-1	-2	
C_2	0.5	1	1.5		1	0	-1	
C_3	0.333	0.667	1		2	1	0	
	Matrix *M*							
	C_1	C_2	C_3		C_1	C_2	C_3	
C_1	1	5	9		1	5	9	
C_2	0.2	1	3		0.2	1	5	
C_3	0.11	0.33	1		0.11	0.2	1	
Sum	1.31	6.33	13		1.31	6.2	15	
	Matrix *M**							
	C_1	C_2	C_3	[*W*]	C_1	C_2	C_3	[*W*]
C_1	0.76	0.79	0.69	0.75	0.76	0.79	0.69	0.72
C_2	0.15	0.16	0.23	0.18	0.15	0.16	0.23	0.22
C_3	0.08	0.05	0.08	0.07	0.08	0.05	0.08	0.06
Sum	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

The consistency test for case #1 produces the results shown in Table 4. In this case, for n = 3, the random consistency index to be applied is 0.52, which produces the consistency ratios (r_c) shown in the last row of Table 4. Since $r_c < 0.1$ for option A, the matrix *M* is consistent, $r_c \geq 0.1$ for option B, therefore, there is inconsistency in *M*.

Table 4
Consistency test of the hypothetical case #1 for options A and B

	Option A			Option B		
	C_1	C_2	C_3	C_1	C_2	C_3
[*K*]	3.065	3.017	3.005	3.254	3.089	3.017
λ_{max}	3.065			3.254		
i_c	0.032			0.127		
r_c	**0.062**			**0.244**		

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The same procedure was followed for the other 12 cases. Table 5 shows the priority vectors (criteria weights) of the 13 hypothetical cases for options A and B in Table 1. Also shown is the vector of weights calculated by the ROCW approach using Equation 8. The standard deviations of the weights for each case are presented graphically in Figure 3. Taking the values of the ROCW method as a reference, the *MRE* and *RMSE* estimates for the 13 cases are shown in Figures 4 and 5, respectively.

Table 5
Priority vectors of the 13 hypothetical cases

Case	Option	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9	C_{10}	C_{11}	C_{12}	C_{13}	C_{14}	C_{15}
1	A	0.748	0.180	0.071												
	B	0.723	0.216	0.061												
	ROCW	0.611	0.278	0.111												
2	A	0.633	0.207	0.100	0.060											
	B	0.595	0.254	0.107	0.044											
	ROCW	0.521	0.271	0.146	0.063											
3	A	0.547	0.211	0.115	0.074	0.053										
	B	0.503	0.260	0.134	0.068	0.035										
	ROCW	0.457	0.257	0.157	0.090	0.040										
4	A	0.481	0.205	0.121	0.083	0.061	0.048									
	B	0.434	0.254	0.148	0.086	0.049	0.029									
	ROCW	0.408	0.242	0.158	0.103	0.061	0.028									
5	A	0.429	0.197	0.123	0.087	0.067	0.054	0.045								
	B	0.382	0.243	0.154	0.097	0.061	0.038	0.024								
	ROCW	0.370	0.228	0.156	0.109	0.073	0.044	0.020								
6	A	0.388	0.187	0.121	0.089	0.069	0.057	0.048	0.042							
	B	0.340	0.231	0.156	0.105	0.070	0.046	0.031	0.021							
	ROCW	0.340	0.215	0.152	0.111	0.079	0.054	0.033	0.016							
7	A	0.353	0.177	0.118	0.088	0.071	0.059	0.050	0.044	0.039						
	B	0.307	0.218	0.154	0.109	0.076	0.053	0.037	0.026	0.019						
	ROCW	0.314	0.203	0.148	0.111	0.083	0.061	0.042	0.026	0.012						
8	A	0.325	0.167	0.114	0.087	0.071	0.060	0.052	0.046	0.041	0.037					
	B	0.279	0.206	0.151	0.111	0.081	0.059	0.042	0.031	0.022	0.017					
	ROCW	0.293	0.193	0.143	0.110	0.085	0.065	0.048	0.034	0.021	0.010					
9	A	0.301	0.158	0.110	0.085	0.070	0.060	0.052	0.047	0.042	0.038	0.035				
	B	0.256	0.195	0.147	0.111	0.084	0.063	0.047	0.035	0.026	0.020	0.015				
	ROCW	0.275	0.184	0.138	0.108	0.085	0.067	0.052	0.039	0.027	0.017	0.008				
10	A	0.281	0.150	0.106	0.083	0.069	0.060	0.052	0.047	0.043	0.039	0.036	0.034			
	B	0.237	0.185	0.143	0.111	0.086	0.066	0.051	0.039	0.030	0.023	0.018	0.014			
	ROCW	0.259	0.175	0.134	0.106	0.085	0.068	0.054	0.043	0.032	0.023	0.015	0.007			
11	A	0.263	0.143	0.102	0.081	0.068	0.059	0.052	0.047	0.043	0.040	0.037	0.034	0.032		
	B	0.220	0.175	0.139	0.110	0.086	0.068	0.053	0.042	0.033	0.025	0.020	0.016	0.013		
	ROCW	0.245	0.168	0.129	0.104	0.084	0.069	0.056	0.045	0.036	0.027	0.019	0.012	0.006		
12	A	0.247	0.136	0.098	0.078	0.066	0.058	0.052	0.047	0.043	0.040	0.037	0.035	0.033	0.031	
	B	0.205	0.166	0.134	0.108	0.087	0.069	0.056	0.044	0.035	0.028	0.022	0.018	0.015	0.012	
	ROCW	0.232	0.161	0.125	0.101	0.083	0.069	0.057	0.047	0.038	0.030	0.023	0.017	0.011	0.005	
13	A	0.242	0.132	0.095	0.076	0.064	0.056	0.050	0.045	0.042	0.038	0.036	0.034	0.032	0.030	0.028
	B	0.193	0.158	0.130	0.106	0.086	0.070	0.057	0.046	0.038	0.030	0.025	0.020	0.016	0.013	0.011
	ROCW	0.221	0.155	0.121	0.099	0.082	0.069	0.058	0.048	0.040	0.033	0.026	0.020	0.014	0.009	0.004

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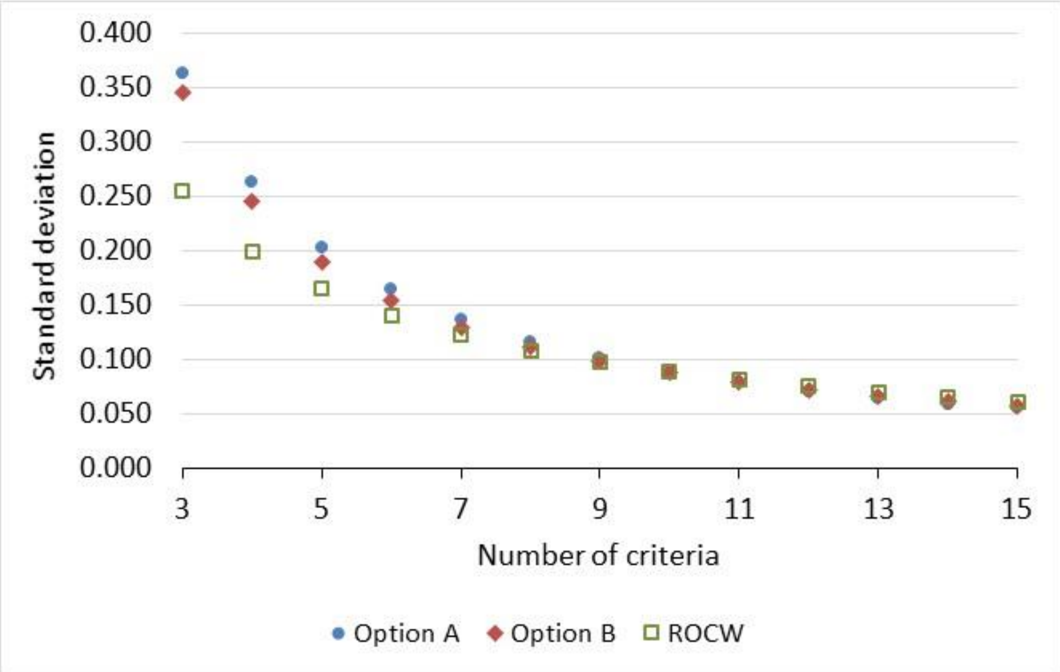

Figure 3 Standard deviations of weights obtained through options A and B and the ROCW approach

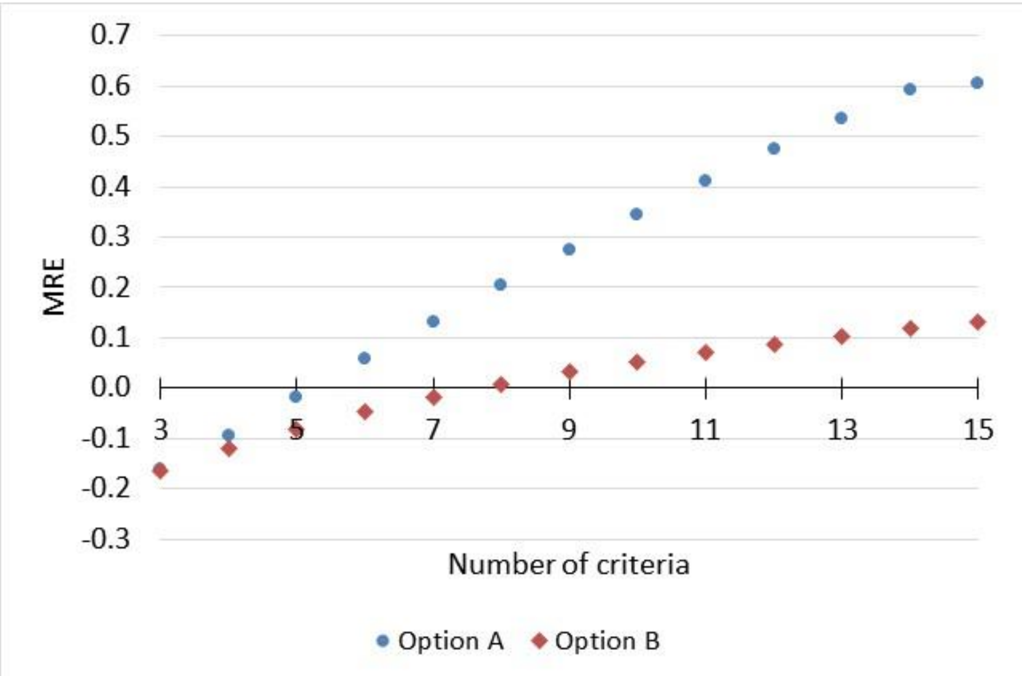

Figure 4 Mean relative error (*MRE*) of priority vector of options A and B compared to that of ROCW for the 13 hypothetical cases.

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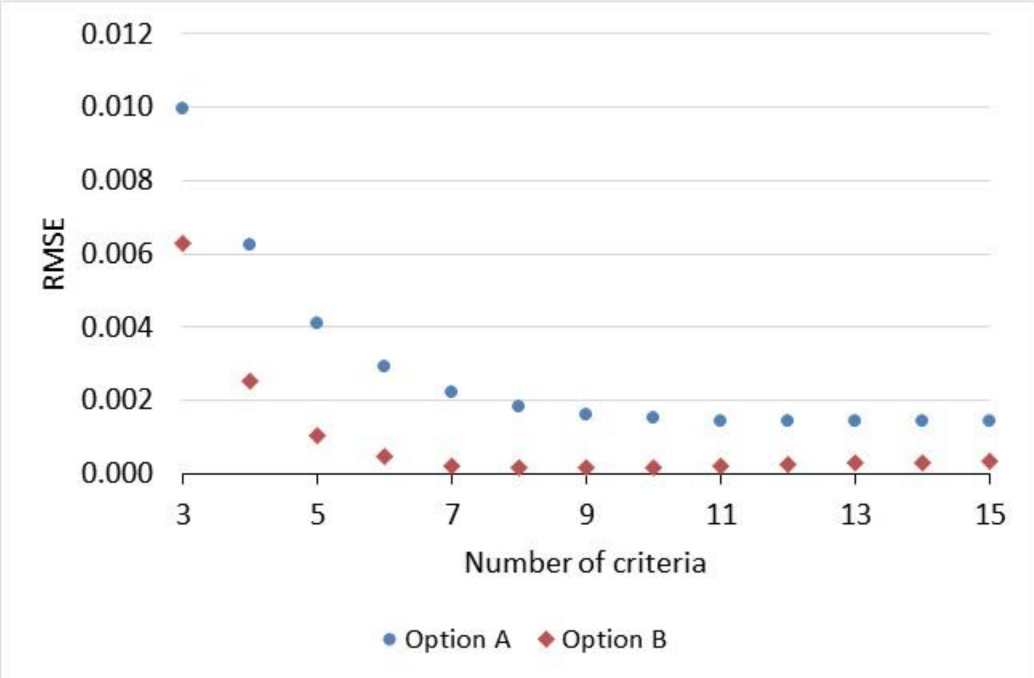

Figure 5 Root mean square error (*RMSE*) of priority vector of options A and B compared to that of ROCW for the 13 hypothetical cases

Finally, the consistency ratios for all cases are shown in Figure 6, along with the consistency threshold.

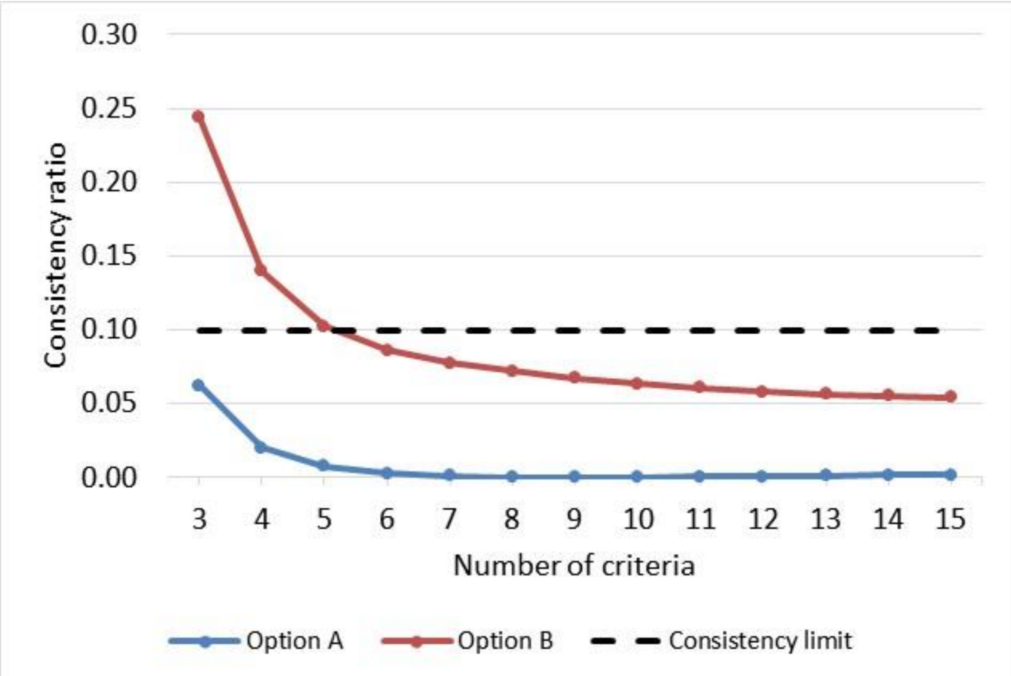

Figure 6 Consistency ratios of the comparison matrices from options A and B

3.2. Example from literature

The example chosen to apply the above method corresponds to the problem of selecting a contractor for the implementation phase of a construction project. This problem is common in the decision-making of a project manager (Al-Harbi, 2001). In this article, the following six criteria for the prequalification of potential contractors were considered: experience (C_1), financial stability (C_2), quality performance (C_3), manpower resources (C_4), equipment resources (C_5), and current workload (C_6). The pairwise comparison matrix of these criteria was built using Expert Choice, a commercial software that simplifies the application of the AHP but does not exclude the participation of a group of experts in the decision-making process (Al-Harbi, 2001). The comparison matrix *M* of these criteria and the priority vector *W* published in the article are shown in Table 6. This matrix *M* passed the consistency test, since $r_c = 0.05 < 0.1$.

Table 6
Pairwise comparison matrix of criteria (*M*) and priority vector (*W*) (Al-Harbi, 2001)

	C_1	C_2	C_3	C_4	C_5	C_6	$[W]$
C_1	1	2	3	6	6	5	0.372
C_2	0.5	1	3	6	6	5	0.293
C_3	0.667	0.667	1	4	4	3	0.156
C_4	0.333	0.333	0.25	1	2	0.5	0.053
C_5	0.333	0.333	0.25	0.5	1	0.25	0.039
C_6	0.2	0.2	0.667	2	4	1	0.087

* $\lambda_{max} = 6.31$, $i_c = 0.062$, $i_r = 1.24$, $r_c = 0.05$

Table 6 shows that the order of priority of the six criteria in this example is: $C_1 > C_2 > C_3 > C_6 > C_4 > C_5$. Therefore, they can be assigned an order number as shown in Table 7.

Table 7
Order of the six criteria for the example

Criteria	Order
C_1	1
C_2	2
C_3	3
C_4	5
C_5	6
C_6	4

By comparing these criteria with their order numbers according to options A and B, we can develop the matrix *M'* based on the order of the criteria. Then, the matrix *M'* is transformed into the matrix *M* on the scale 1-9, and subsequently it is normalized to obtain the matrix *M** and the priority vectors *W*. The resulting matrices are tabulated in Table 8. The matrix *M* is consistent since $r_c < 0.1$ in both options.

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Table 8
Application of the proposed method to the example for options A and B

	Option A [a]							Option B [b]						
	Matrix *M'*													
	C_1	C_2	C_3	C_4	C_5	C_6		C_1	C_2	C_3	C_4	C_5	C_6	
C_1	1	2	3	5	6	4		0	-1	-2	-4	-5	-3	
C_2	0.5	1	1.5	2.5	3	2		1	0	-1	-3	-4	-2	
C_3	0.333	0.667	1	1.667	2	1.333		2	1	0	-2	-3	-1	
C_4	0.2	0.4	0.6	1	1.2	0.8		4	3	2	0	-1	1	
C_5	0.167	0.333	0.5	0.833	1	0.667		5	4	3	1	0	2	
C_6	0.25	0.5	0.75	1.25	1.5	1		3	2	1	-1	-2	0	
	Matrix *M*													
	C_1	C_2	C_3	C_4	C_5	C_6		C_1	C_2	C_3	C_4	C_5	C_6	
C_1	1	2.6	4.2	7.4	9	5.8		1	2.6	4.2	7.4	9	5.8	
C_2	0.385	1	1.8	3.4	4.2	2.6		0.385	1	2.6	5.8	7.4	4.2	
C_3	0.238	0.556	1	2.067	2.6	1.533		0.238	0.385	1	4.2	5.8	2.6	
C_4	0.135	0.294	0.484	1	1.32	0.714		0.135	0.172	0.238	1	2.6	0.385	
C_5	0.111	0.238	0.385	0.758	1	0.556		0.111	0.135	0.172	0.385	1	0.238	
C_6	0.172	0.385	0.652	1.4	1.8	1		0.172	0.238	0.385	2.6	4.2	1	
Sum	2.04	5.07	8.52	16	19.9	12.2		2.04	4.53	8.60	21.38	30.00	14.22	
	Matrix *M**													
	C_1	C_2	C_3	C_4	C_5	C_6	[*W*]	C_1	C_2	C_3	C_4	C_5	C_6	[*W*]
C_1	0.49	0.51	0.49	0.46	0.45	0.48	0.48	0.49	0.57	0.49	0.35	0.30	0.41	0.43
C_2	0.19	0.20	0.21	0.21	0.21	0.21	0.21	0.19	0.22	0.30	0.27	0.25	0.30	0.25
C_3	0.12	0.11	0.12	0.13	0.13	0.13	0.12	0.12	0.08	0.12	0.20	0.19	0.18	0.15
C_4	0.07	0.06	0.06	0.06	0.07	0.06	0.06	0.07	0.04	0.03	0.05	0.09	0.03	0.05
C_5	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.03	0.02	0.02	0.03	0.02	0.03
C_6	0.08	0.08	0.08	0.09	0.09	0.08	0.08	0.08	0.05	0.04	0.12	0.14	0.07	0.09
Sum	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

[a] $\lambda_{max} = 6.02$, $i_c = 0.004$, $i_r = 1.25$, $r_c = 0.003$; [b] $\lambda_{max} = 6.54$, $i_c = 0.108$, $i_r = 1.25$, $r_c = 0.086$

The standard deviation of the priority vectors *W* obtained (Table 8) with respect to the vector *W* of the bibliographic source (last column in Table 6) is equal to 0.144 for option A and 0.139 for option B. The absolute errors of each element of *W* are shown in Figure 7. The mean relative error is 0.02 for option A and -0.06 for option B, and the root mean square error is 0.0035 and 0.0009, respectively.

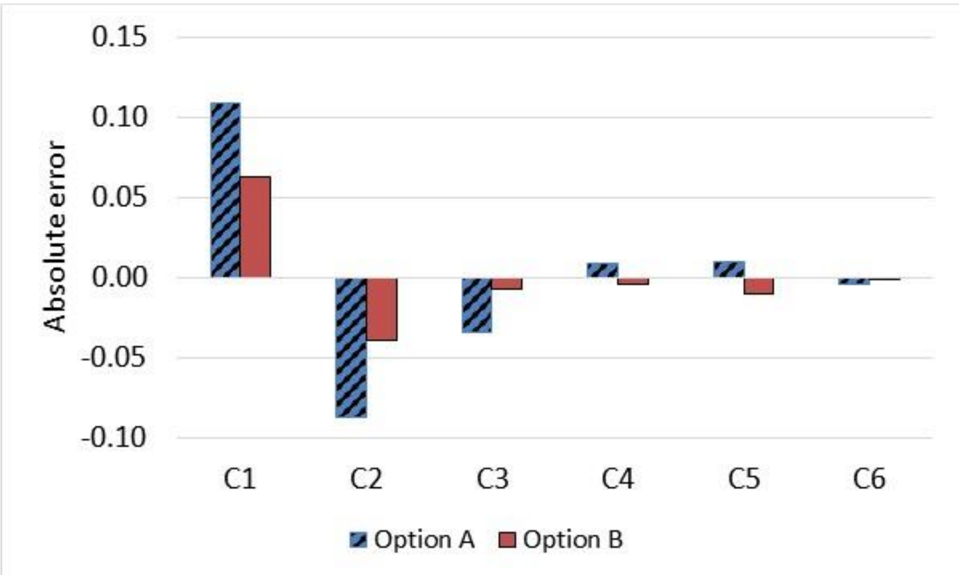

Figure 7 Absolute errors of the six elements of the priority vector from options A and B for the example

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4. Discussion

4.1. Hypothetical cases

The study of the hypothetical cases shows that, in general, the standard deviation (σ) of the coefficients of the *W* vector, i.e., the criteria weights, of the AHP method decreases as the number of criteria increases (Figure 3). This is logical because the weights (values < 1 and sum of the *n* weights equal to 1) have a lower value as their number increases. However, if we compare the results for options A and B of the proposed method, it turns out that σ is higher in option A in cases #1 to #8. But, σ is very similar from $n > 10$ in both options. In the case of ROCW, σ is even lower than in the case of option B, but when $n > 10$, it becomes similar to that of the two options.

Comparing the results of the proposed method with those of the ROCW approach, shows that in all cases option B offers a lower *MRE* (Figure 4) and *RMSE* (Figure 5) than option A. Moreover, its *RMSE* values are very close to 0 for the cases where $n \geq 6$. In terms of matrix consistency, as seen in Figure 6, option B is not acceptable when $n < 6$, since $r_c \geq 0.1$. Moreover, in all cases, option A provides lower r_c values than option B. Nevertheless, the use of option B cannot be excluded when $n \geq 6$, as can be seen below from the example studied.

4.2. Example from literature

According to the proposed method, the first step is to rank the criteria in order of importance. The ranking of the six criteria in the example studied, as shown in Table 7, was relatively easy in this case, since the final weights obtained by these criteria were known. However, if the vector *W* had not been available, it would certainly have been easier to determine the order of importance of the six criteria than to construct the *M* matrix in Table 6 directly by comparing the criteria according to the AHP 1-9 scale (Figure 1).

It should be noted that the value of i_r for $n = 6$ in the published example (1.24) is slightly different from that used in our application of the method (1.25), although the consistency test is passed in both cases. The reason for this difference is that i_r is represented by average i_c values obtained from a random simulation of the pairwise comparison vectors *K*. As indicated in Franek and Kresta (2014), in the first case the value of i_r was obtained from 500 simulations, while in our case the value of ir corresponds to about 500 000 simulations. More detailed information can be found in Franek and Kresta (2014).

The weights of the criteria (vector *W*) obtained by the proposed method in both options reproduce the established order of the criteria. Overall, option B offers a lower standard deviation than option A. In addition, individually, all the elements of its vector *W* show lower deviation than those of option A with respect to the published values. It can be said that for this example, globally, option A tends to slightly overestimate the criteria weights, while option B underestimates them, with an average relative deviation of +2% and -6%, respectively. However, we consider that option B makes more sense because the difference between the orders of two criteria does not depend on the equidistance between them. For example, according to option B, the difference between the orders of C_3 (3^{rd}) and C_4 (5^{th}) is -2, which is the same as that between C_6 (4^{th}) and C_5 (6^{th}). On the contrary, the ratio between C_3 (3^{rd}) and C_4 (5^{th}) is 3/5, which is different from the ratio between C_6 (4^{th}) and C_5 (6^{th}), which is 4/6. Both options present a very low *RMSE* (lower in the case of option B), so it can be said that the result of the proposed method is sufficiently close to that estimated by the classical AHP method.

The method is applicable to problems where there is a chain of criteria that can be ranked in order of priority with a single ranking number per criterion. In the case where several criteria have the same priority, it is recommended to group them into a single criterion and then distribute the weight among the criteria of equal importance. For example, in the case of six criteria, if the order were $C_1 = C_2 > C_3 = C_4 > C_5 = C_6$, the method for three criteria can be applied: $C_1 > C_3 > C_5$ and then the resulting weights of C_1, C_3 and C_5 divided by two. But, in general, in the case of several criteria with equal priority, we can calculate the resulting weight w_i (i = 1, …, n) of criterion C_i by means of the following expression:

$$w_i = \frac{w'_j}{\sum_{j=1}^{n'} w'_j \cdot k_j} \tag{11}$$

Where w'_j represents the weight of the proxy criterion C'j (j = 1, …, n') and k_j the number of criteria with the same priority as the criterion C_i.

For example, in the case of five criteria $C_1 = C_2 = C_3 > C_4 > C_5$ (n = 5), the proposed method would be applied to three criteria $C'_1 > C'_2 > C'_3$ (n' = 3), where k_1=3, k_2=1 and k_3=1, being C'_1 a proxy of C_1, C_2 or C_3; C'_2 a proxy of C_4, and C'_3 a proxy of C_5. However, the subsequent comparison of alternatives using the AHP method should be done using the original C_i criteria, regardless of the weight of those criteria.

4.3 Considerations for criteria comparison

The main difference between the proposal introduced in the present work and the AHP method lies in criteria comparison, performed through an ordering of criteria here, while a set of pairwise comparisons is to be carried out in the canonical technique. Although this pairwise comparison working mode was justified by Saaty (2008) by the fact that comparing two features is easier than proposing an ordering of many of them. However, the quantification of the priority of a criterion over another one is not only difficult to achieve, but may also lead to undesired rankings.

First, if an alternative can be evaluated in terms of criteria from different areas (including for instance economic profitability, environmental impact, technical considerations and social benefits), ordering these criteria may be cumbersome, but quantifying the priorities in the 1-9 scale might be impossible. In addition, a DM may not have expertise in all these areas and, therefore, might not be able to appropriately rate his/her preferences, or only intuitively rate them (which may have significant consequences on the final criteria weights). In these cases, and as stated in Munier and Hontoria (2021), "[the] DM should be a multiple technical expert" to be able to comprehensively compare criteria numerically.

This observation might be amplified in the case of projects involving population. For instance, in the framework of isolated community electrification (Ponsich et al., 2022), several configurations of the electrification systems can be proposed to end-users (i.e., community members), representing different alternatives in terms of investment cost, supplied energy and available power. When asked to evaluate their priorities, populations that are not used to electricity services were barely able to understand the abstract concepts of energy and power, and it was even more difficult for them to compare and, in addition, evaluate numerically the magnitude of their

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preferences. In this case, a simple ordering may be easier to achieve a comprehensive prioritization of criteria.

When priorities are set and quantified in an approximate way, they might result in inappropriate relationships between criteria. For instance, consider a DM having to compare three criteria, C_1, C_2 and C_3. The DM intuitively feels that he/she slightly prefers C_1 over C_2, and slightly prefers C_2 over C_3. Then, the 1-9 scale forces him/her, as a minimal option, to set C_1 as twice as important as C_2 and C_2 as twice as important as C_3. Therefore, C_1 is implicitly four times more important than C_3, whereas the DM still has a slight preference regarding C_1 over C_3. It is clear that this would not happen when using the simple ordering proposed in our work.

Finally, considering the inherent issues mentioned for the AHP (for complex problems, the high number of comparisons to be quantitatively performed often makes experts feel fatigued and lose interest) in addition to the previously mentioned problems, the approach proposed here may show, in some cases, practical benefits over the classical pairwise comparison on a 1-9 scale.

5. Conclusions

The construction of the pairwise comparison matrix *M* is the most critical point of the AHP method. Its construction usually requires the assistance of a group of experts to make value judgments among the criteria considered necessary for evaluating alternatives. These judgments are not free from a certain degree of subjectivity.

The method presented here is intended to facilitate the pairwise comparison for the construction of the *M* matrix of the AHP, thereby reducing the degree of subjectivity. The only step that requires human intervention is the ranking of the criteria. The rest of the steps can be programmed, e.g., in a spreadsheet, and performed automatically and objectively.

The method proposed in both options differs from existing methods based on the estimation of the criteria weights from the ordinal numbers of the order of importance, such as the ROCW method, in that the calculation of weights and the consistency test are performed according to the AHP method. That is, the method only modifies the construction of the pairwise comparison matrix *M* with the help of Equations 2 (option A) or 3 (option B), but does not change the rest of the AHP. In addition, this method is applicable even when several criteria have the same order of priority, if the correction indicated by Equation 11 is applied to the weights calculated for the proxy criteria.

Although the proposed method may produce a matrix M different from the one obtained by the classical method, the order of priority of the criteria is preserved. Considering that the “true” weights of criteria are unknown in practice (Roszkowska, 2013), the proposed method has been shown to produce results comparable to the ROCW approach, especially in option B for $n \geq 6$.

Both options of the method give reasonable results, but we believe that it is more advisable to use option B based on differences of criteria rankings for problems with more than five criteria. Option A based on criteria ranking ratios would be the most appropriate for problems with $n < 6$ because it provides consistency in the *M* matrix.

The work focused on establishing the weights of the criteria, but the method presented is obviously also applicable to the selection of alternatives.

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