Fuzzy Cash Flow Analysis of Fluctuations in Interest Rates at the State Bank of Pakistan using Present Worth Criterion

Ali Gohar
College of Business and Management,
Fahad Bin Sultan University,
Tabuk, Saudia Arabia
Email: agohar@fbsu.edu.sa

ABSTRACT

A systematic approach is required to analyze present worth by incorporating annual interest rate fluctuations. This is a key factor in attracting local and foreign investors and maintaining a level of economic advancement. This study primarily focuses on the domain of the State Bank of Pakistan (SBP) and contributes towards achieving three major objectives. The first objective includes modeling interest rate values as triangular fuzzy numbers, and using the Analytic Hierarchy Process (AHP), a Multi-criteria Decision-Making technique (MCDM). Second, the triangular fuzzy interest rates are forecasted over a future timespan. Subsequently, the last objective includes the use of the Present Worth Analysis (PWA) to calculate the final net present value. The results demonstrate a trend in interest rates with net present values of specific future years. This research is a good initiative for the SBP and helpful for both foreign and local investors. The study is novel in that it involves the merging of qualitative and quantitative data. Furthermore, the combination of the forecasting technique, the AHP, fuzzy set theory, and present worth criteria renders a unique approach to formulating present worth. There is no previous research of this magnitude that has been conducted on this topic in a developing country such as Pakistan. This study delves into the annual interest rate fluctuations, providing valuable insights for present worth calculations.

Keywords: SBP; fuzzy; MCDM; AHP; PWA; interest rates

1. Introduction

Currently, Pakistan is seeing an observable shift in economic organizations and financial structures. The country has witnessed a long list of failed companies that practiced poor decision-making methods. Companies such as Bhoja Airlines and Aero Asia were not able to decide on how to invest to upgrade technology (Hasan, 2016). Moreover, banks such as NIB have recently closed due to huge losses incurred. Alternatively, many investors are looking for favorable monetary terms and economic stability in order to invest a principal amount or to start a company (Ajaz, et al., 2022). Pakistan has long employed efforts to attract local and foreign investors. These investors directly or indirectly come into contact with the State Bank of Pakistan (SBP) which manages the country's economy and generates any fiscal policies according to the current regime.

Annual interest rates for the country are also formulated considering the current economic state of the country (Insight Securities, 2022).

The SBP is the central bank in Pakistan and plays a vital role in making financial policies and dealing with fluctuations in interest rates by considering increases or decreases in inflation, unemployment, etc. The state bank also facilitates projects related to infrastructure, housing loans, renewable energy schemes, etc. It also facilitates small and medium enterprises (SMEs), microfinance, agriculture financing, guidelines on risk management, commercial paper securitization, etc. This bank is solely responsible for making the currency used in citizen's daily lives (Mazhar & Rehman, 2021). However, due to several national issues, the interest rate does not follow a consistent trend. Every year, fluctuations in interest rates are observed, which has a direct impact on the country's economy. It is worth mentioning that the fluctuations in the interest rate of the SBP from 2015-2020 have had a great impact on the decisions of investors (Rafiq, Jun, Naseem, & Mohsin, 2019). Some of the reasons for this trend of fluctuation are included in Figure 1.

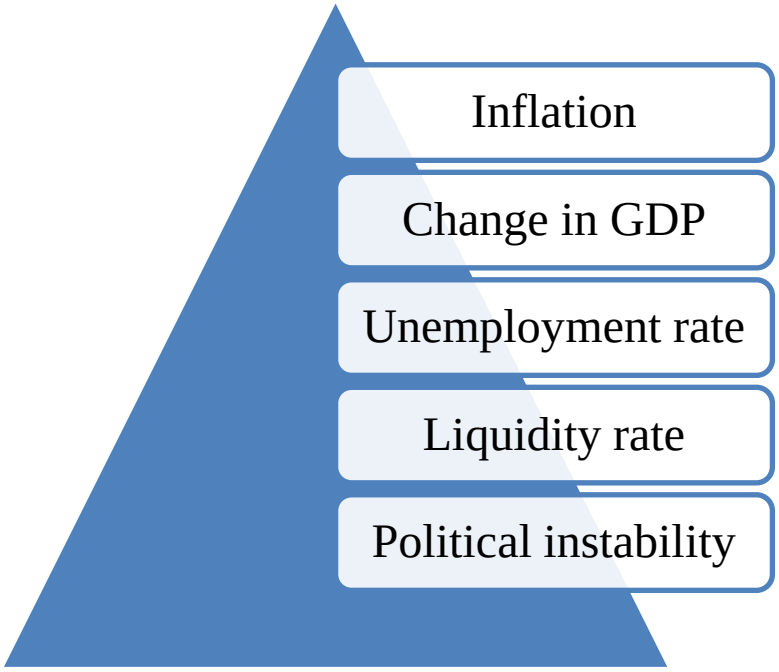

Figure 1 Reasons for the fluctuations in the trend in the interest rates.
Source: Author's creation

Furthermore, as the interest rate decreases, more people can borrow money from banks and since this allows more people to have more money, inflation will increase. As inflation increases, the interest rate increases so that the inflation will then decrease. Therefore, there is an indirect relationship at the beginning of this unending loop of changes in inflation rates and the corresponding interest rates in the economy (Angelina & Nugraha, 2020).

Similarly, a change in GDP accounts for a change in an economy where an increase in GDP means the economy is growing and businesses are doing well. Therefore, as GDP

increases the unemployment rate decreases. The lower unemployment rate means more investment and more people being employed and earning money. As a result, demand becomes greater than supply causing an increase in inflation. Inflation causes problems such as panic and hunger among the population. This causes an increase in interest rates which leads to less investment and people losing their jobs which increases the unemployment rates. As the unemployment rate increases, inflation decreases and interest rates decrease which in turn reduces unemployment over time. This cycle keeps repeating itself and demonstrates the importance of decision-making in financial institutions (Krustev, 2019).

Effective methods and models that provide information about trends in interest rate fluctuations at the State Bank of Pakistan as well as useful forecasting techniques can provide solutions to uncertainty among investors. New techniques have been developed relating to commercial systems and the latest emerging business techniques. Moreover, rapidly changing fiscal policies and fluctuating interest rates have given rise to the development of newer forecasting, predicting, and estimation tools. These forecasting techniques have both advantages and disadvantages. The main benefit is the assistance in decision making, whereas the main disadvantage is the inaccuracy and imprecision inherent within each respective technique (Wang & Jiang, 2019). Furthermore, a vital part of monetary establishments in Pakistan is the decision-making process with the use of different decision matrices and procedures. Decision-making is imperative for the achievement of goals, motivation of employees, utilization of resources, and many more aspects for the improvement of commerce in the local industry (Martínez-Peláez, et al., 2023). Decisions regarding economic models are based on different features which include expert opinions, historical data, political regimes, and finance measuring institutions. Each of these features takes into consideration some sort of objective data. The ease of availability of such well-interpreted factual data determines the decision procedures (Abel, et al., 2021). Unfortunately, one can hardly find such well-constructed information resulting in difficult judgments being made. Such cases result in poor decision modeling and failed institutions.

Decision-making is one of the core components of management, in both routine habits in an individual's daily life and in business organizations where decision-making uses fully developed professional procedures. Competition in current market systems and the latest advanced technologies provide multiple alternatives for decision-making, which makes the process complex. Decision-making methods are specialized procedures that deal with these types of difficult situations (Khan & Ali, 2020). For a situation involving multiple criteria, Multiple Criteria Decision Making (MCDM) methods are used. These methods allow for the selection of the best alternative from among several options, taking into consideration the requirements of the user. They also allow the development of cause-and-effect weightage systems to decide which criteria possess an effective role (Abdullah, et al., 2021). This type of decision-making method allows the most critical causes for changing interest rates from among the stated reasons to be determined. One of the best approaches among multiple criteria decision-making methods is the Analytical Hierarchy Process (AHP). This method quantifies the intensity of the importance of each criterion. Individual weights are assigned based on expert opinions and user public surveys; the weights assigned are then compared with each other, and a final weighted decision matrix that ranks the criteria according to compared weights is constructed. The criteria can then further be applied to the alternatives where the best possible alternative

can be determined (Khan & Ali, 2020). This eases the decision-making process and provides substantial assistance for business organizations, managers, and stakeholders to make decisions. As a result, these decision-making methods have paramount importance in making key decisions in an organization to maximize profits. Such models help managers, as well as engineers accomplish operational objectives and in turn, are valuable in achieving overall industrial aims and objectives (Nguyen et al., 2023).

The main focus of this article is to develop a model to account for fluctuations in interest rates at the SBP based on several criteria by quantifying each criterion using expert opinion and multi-criteria decision modeling. Then, the model can be used to forecast future interest rates. The technique, i.e., present worth criterion, is applied with the use of triangular fuzzy quantification, which acts as a calculator for future worth, keeping in mind the fluctuation trend. This model provides powerful tools that are helpful to businesses and can define a trend for fluctuations in SBP's interest rate.

2. Scope of research

The scope of the research ranges from providing a model that will offer guidance for future investments of the SBP to implementing the MCDM technique, forecasting the interest rates, and using the triangular fuzzy approach to evaluate the net present value in each subsequent year. The research will greatly assist decision-making with the current interest rates in mind. Furthermore, it will motivate investors to invest in the future, injecting more money in the country's economy. The proposed approach will also lead to an increase in the business growth index, generating more employment and producing more goods and services, and also filling the gap of research on this topic in the case of a developing country like Pakistan.

3. Literature review

The stability of Pakistan's economy has long been in a state of turmoil and perplexity. The fragile state of administration has proved powerless to generate effective monetary policies. A poor political regime together with swiftly changing fiscal policies has caused considerable changes and inconsistency in interest rates. These circumstances require drastic measures to be taken to regulate the country's economy (Ul Mustafa, Abro, Hussain, & Ali, 2021). An increase in local and foreign investments is one of the most imperative solutions to such an escalating problem.

The literature includes many online articles and reports related to the monetary policies and economic accomplishments of the State Bank of Pakistan (SBP). The official SBP website states that there are currently 103 working publications of the SBP (State Bank of Pakistan, 2016). These reports include all the current ongoing press releases which cover the latest news relating to monetary policies. Furthermore, they also cover important events and public awareness warnings about economic issues. Moreover, the data section of the reports deals with interest rates every year, and the guidelines according to changing market trends are available for investors who want to set up a business plan or commercial bank in Pakistan. The data from these reports was useful in generating a secondary data source for the current study. It provided in-depth knowledge about factors affecting interest rates in Pakistan. Moreover, secondary data were calculated from the

study, which were more understandable and based on facts (State Bank of Pakistan, 2016). To formulate a standard evaluation, criterion, and techniques, a rigorous literature review was conducted to ensure that the research requirements and its gaps were met accordingly.

Keeping the aforementioned discussion in mind, our goal is to develop economic techniques to help create a competent solution for attracting investors. These techniques include different forecasting methods, trend settings for fluctuating interest rates, and present-worth cash flows. This section highlights the main research conducted to develop these techniques in Pakistan, as well as on a global scale. The inclusion of a literature review from international sources can benefit Pakistan by finding proficient techniques. As a result, a wider approach was utilized for the literature review.

Chiu and Park (1994) presented an analysis of cash flows using present worth analysis. The results were produced using a fuzzy analysis approach, particularly the triangular fuzzy method. It was the first-ever study to present an alternative to the conventional cash flow models in the form of engineering an economic analysis that rules out vague human thoughts. The uncertainties of future cash flows were quantified using fuzzy set theory. The study gives immense significance to replacing vagueness with numerical values.

Another similar study carried out a cash flow analysis using the fuzzy present worth analysis model. This study implemented the extended fuzzy version of the present worth analysis to devise an application for a water treatment device in a local municipality (Boltürk & Kahraman, 2022). Furthermore, the uncertain cash flows were modeled as triangular fuzzy numbers. Using the principal amount value, the present worth formulation was derived. A comparison test between actual present-worth cash flows and cash flows obtained from the triangular fuzzy approach was conducted. The use of dummy variables and inculcation of triangular fuzzy sets in the present worth analysis was taken from this source and has thus formed a benchmark for this current research.

Furthermore, an adaptive hierarchical fuzzy logic system for the modeling of financial systems presented detailed research on cash flow fluctuations in Australia (Mohammadian & Kingham, 2004). The research included the use of a fuzzy logic system with genetic algorithms for the prediction of interest rate fluctuations in the country. The ideology can also be integrated into the economic system of Pakistan, with only slight changes in linguistic variables and economic indicators. The main working criteria and outline flowchart are highlighted in Figure 2.

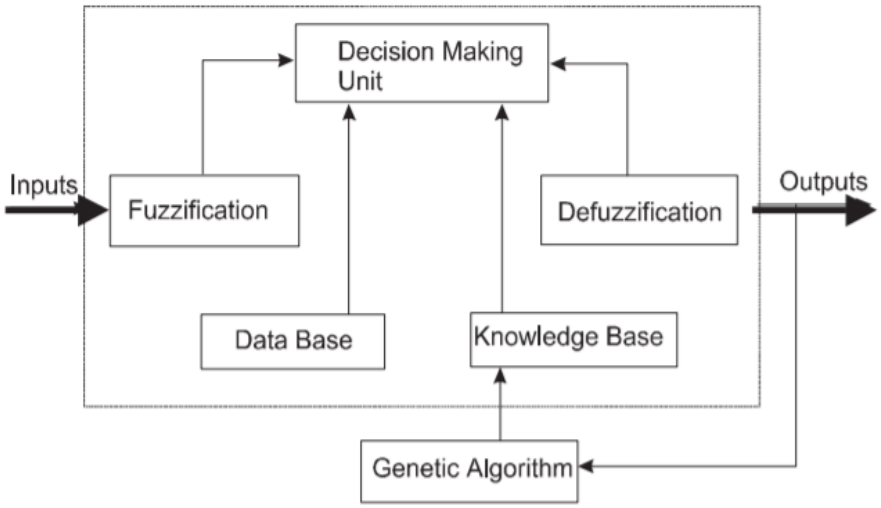

Figure 2 Adaptive hierarchical fuzzy logic system
(Mohammadian & Kingham, 2004)

Figure 2 shows that the inputs and outputs include several economic indicators such as inflation, unemployment rate, and other finance-impacting factors. Mohammadian and Kingham (2004) used a hierarchical system that was introduced by splitting economic indicators into several groups. This division was created based on the influence of each indicator. A fuzzy knowledge base was generated which was forwarded into a genetic algorithm to find solutions. The genetic algorithm is responsible for creating random strings that represent possible solutions for the fuzzy knowledge base. Recent research implemented the use of the hierarchical fuzzy logical system to carry out supplier selection in the case of the oil and gas industry (Sarfaraz, et al., 2023). The research concluded with developing the final hybrid fuzzy and genetic algorithm system and thus the use of the economic indicators and linguistic variables for the current study were derived. Another study involving cash flow analysis using the fuzzy theory comprised a fuzzy zooming model that involved fuzzy arithmetic logic (Berliner & Buehlman, 1993). The study modeled insecure and fluctuating interest rates as fuzzy numbers, and then implemented the fuzzy triangular model to map long-term complex cash flows into simpler periods of cash flows. Using the same technique, the research also introduced an alternative option to unite partial cash flows to give final cash. The generalized approach used by Berliner and Buehlmann (1993) is shown in Table 1. A recent example of the same model can be found in the case of corporate valuation which explores the scenarios beyond the forecasting period (Taliento, 2019).

Table 1
Example of zooming order

Kind of interest rate shock	Original cash flow	1st order zooming	5th order zooming
1st kind	10.49	11.29	10.51
2nd kind	100	78.66	99.89
3rd kind	50.11	78.66	47.51
4th kind	60.39	11.29	62.88

Source: Berliner & Buehlmann (1993)

In addition to the triangular and trapezoidal fuzzy logic system, another technique called the Mamdani fuzzy logic system, which expands the use of a stochastic fuzzy input variable system, was studied. This research is based on a two-step approach as follows: determining cash flow deficits followed by defining stochastic fuzzy variables for deriving the main logic system. The mentioned research provided more controlled outputs with fuzzy logic systems (Bollos, et al., 2019). Similarly, Dimitrovski and Matos (2008) carried out an application of fuzzy set theory in combination with the present worth analysis. The procedure generated answers with the use of crisp values and then modeling fuzzy methods with the same case study. A recent application of this method involved a hybrid combination with spherical fuzzy sets to handle the fuzzy parameters of investments (Boltürk & Seker, 2022).

Financial systems and their forecasting require decision-making in complex scenarios, similar to the focus and the primary approach of the current research study. Different applications of Multi-Criteria Decision-Making (MCDM) techniques include the evaluation of the financial applications from 2000-2018 via multi-attribute value-based and outranking relations methods to justify the usage of ranking-based MCDM models (Marques et al., 2020). A similar application can also be found in the assessment of banking sector companies that require ranking based on their financial indicators via the Best-Worst Method (BWM) and Ranking Alternatives by Perimeter Similarity (RAPS) methods (Alamoudi & Bafail, 2022). Another application involves the use of a rough knowledge-based hybrid MCDM model to plan for the improvement of financial modeling for the life insurance industry. This study tried to bridge the gap between pragmatic financial modeling and soft computing techniques in a dynamic business environment (Shen et al., 2017). Extreme Learning Machine (ELM) based predictor for the forecasting of stock index prices is another application of MCDM techniques, i.e., a hybrid combination of the TOPSIS, PROMETHEE-II, and VIKOR methods (Samal & Dash, 2022). Lastly, one of the most important financial risk predictions is also an application of MCDM techniques, which involved an evaluation of Imbalanced Classification Methods. This study concluded that TOPSIS, an MCDM technique that involves multiple criteria, provided a more reasonable evaluation as compared to a single performance criterion classifier (Song & Peng, 2019).

There are very few applications of the technique under consideration for the current study, i.e., fuzzy AHP for financial forecasting, thus one of the reasons for it being a preferred choice. One of the applications includes the aggregation for a multiple time series prediction. This study utilized Dow Jones Index data for the simulation and showed

that the COVID-19 pandemic had an influence on the time series which adds more value to the forecasting of the Dow Jones Index (Castillo et al., 2022). Financial distress is another issue that was addressed by the Fuzzy AHP framework in the case of an Indian airline, and the study concluded that GDP growth rate and annual inflation have a major influence on its sustainability (Mahtani & Garg, 2018). Similarly, a hybrid combination of Fuzzy AHP and MULTIMOORA methods addressed the financial ratios for financial accounting fraud detection. The study was able to produce comparative results for both methods to enable financial ratio prioritization (Hamal & Senvar, 2022).

Moreover, in the case of Pakistan, the research carried out by Malik & Nishat (2017), provided a degree of volatility in interest rates in Pakistan. This research is in line with the scope of the current study, yet possesses a limited exploration in terms of a detailed evaluation of SBP's fluctuation of interest rates. Based on this discussion, a model to account for the assessment of the interest rates of the SBP, whose fluctuations are based on various factors and criteria was created. For this purpose, the study employed the Fuzzy Analytic Hierarchy Process (FAHP). The use of this method allows the freedom to assess the criteria or factors or to evaluate multiple alternatives based on distinct criteria. This freedom allows for effective and efficient decision-making in complex decision scenarios (Nazim et al., 2022). Its effectiveness can be seen from the various applications in the literature that showcase its involvement in complex decision scenarios such as a combination of hybrid methodologies to evaluate car companies (Ali et al., 2020a), promotion of peace between the two countries, i.e., Pakistan and India (Ali et al., 2020b), and in a hybrid combination with the TOPSIS method to evaluation urea production methods (Ali et al., 2019), to mention a few. Based on the aforementioned discussion, the previous literature includes limited applications of Fuzzy AHP in terms of financial forecasting especially in combination with other techniques. Similarly, research conducted in a developing country such as Pakistan lacks a detailed assessment of fluctuating interest rates, carried out in a hybrid methodological implementation scenario. In summary, the hybrid combination of the techniques being utilized for the current research, i.e., triangular fuzzy numbers, AHP, forecasting of the interest rates, and Present Worth Analysis (PWA), is a unique, first of its kind, comprehensive methodological model.

4. Data collection and methodology

4.1 Data collection

The data collection for the current research study was carried out in two different ways i.e., primary and secondary. The first approach involved obtaining data directly from the State Bank of Pakistan and for that, the authors conducted an interview with an officer in the State Bank of Pakistan about fluctuations in interest rates and used the results from to interview to generate AHP tables for the Triangular Fuzzy Number method. Furthermore, a questionnaire was generated and distributed amongst the officials in the relevant fields to further authenticate the collected data. In the second stage, the official website of the State Bank of Pakistan was used to obtain data on inflation (Statista, n.d.), unemployment rates (World Bank, n.d.a), liquidity rates (CEIC, n.d.), and interest rates (countryeconomy.com, n.d.). The World Bank's report on GDP growth in Pakistan was used as a secondary data source (World Bank, n.d.b). For the primary data collection, 15 experts responded whose profiles are listed in Table 2.

Table 2
Expert profile for the primary data collection

Expert Profile	**Number of the Experts**
Accounting officer	5
Senior Data Analyst	7
Academic Researcher	3

4.2 Methodology

The methodology of the current research is significantly important because of its hybrid approach to cover different aspects of the research in the context of a developing country. The use of such techniques can be justified because methods like the AHP allow the assessment of the criteria factors alone or the evaluation of alternatives based on those distinct criteria. The Consistency Index (CI) determines if the results are sufficiently consistent, i.e., if the CI < 0.1. The utilization of triangular fuzzy numbers removes vagueness from the collected data by assigning numerical values. The data collected from experts is usually in linguistic terms and must be numerically evaluated in order to utilize it for the quantitative assessment. This method allows for the primary and the secondary data to be combined with the human decision. To do so, the current research normalizes the weights collected from the use of the MCDM technique and the ratios from the secondary data, and then multiplies them together to evaluate the rankings, thus removing the vagueness from the results and ensuring data validation.

The methodology of the current research is shown in Figures 3 and 4 in a flow chart.

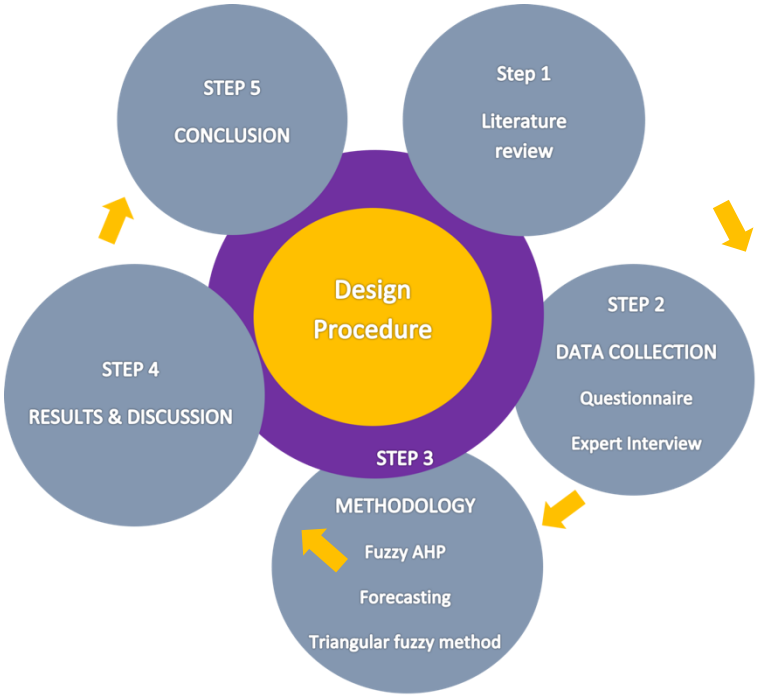

Figure 3 Process flow diagram for the current study

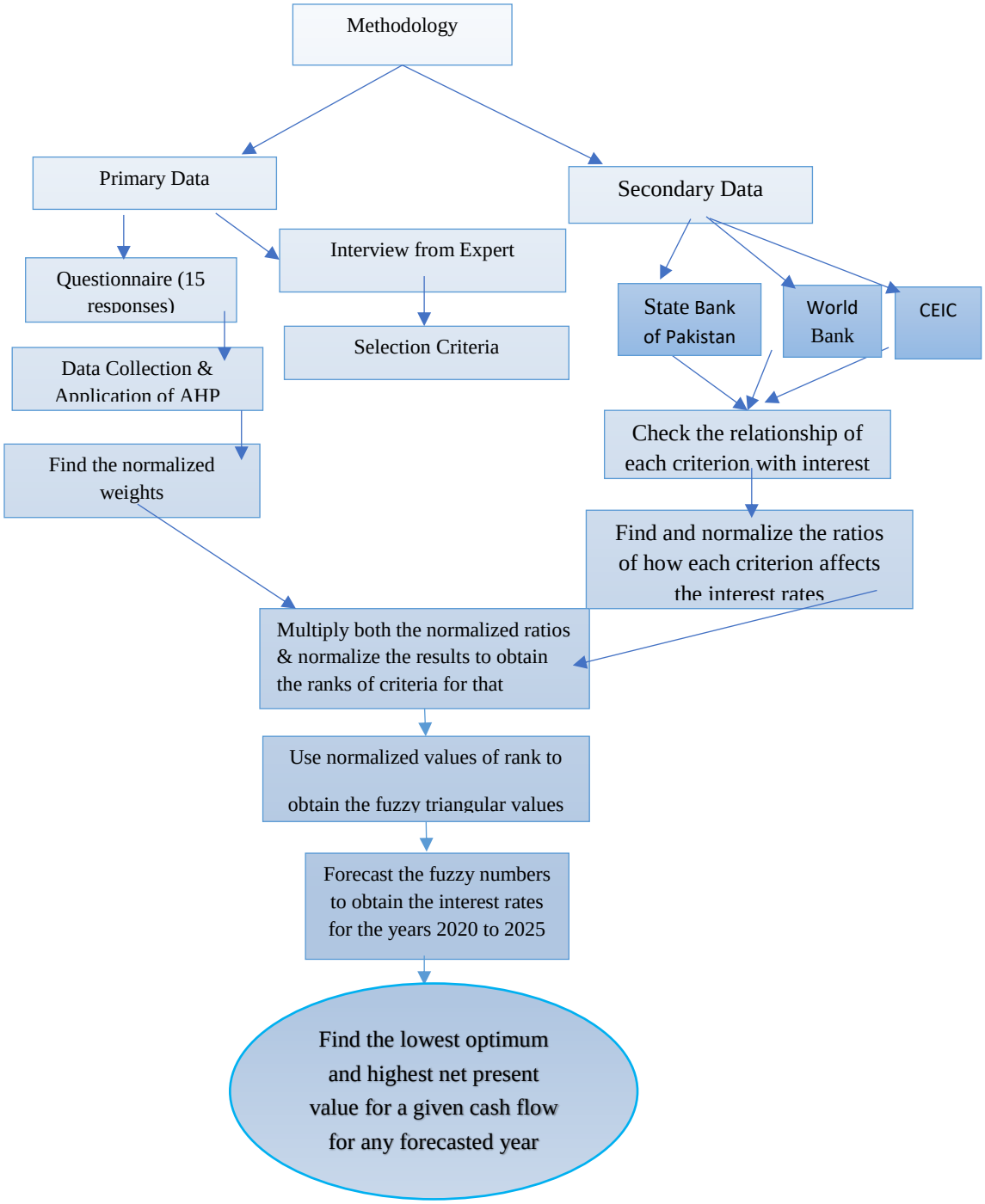

Figure 4 Methodology

4.2.1 Fuzzy Analytical Hierarchy Process (AHP)

The first step in the Analytic Hierarchy Process (AHP) is to determine important criteria that affect the interest rate of a state bank. The technique was introduced by Thomas Saaty (Saaty & Vargas, 1979). For this study, five major criteria were selected which are depicted in Figure 1. The next step involved constructing a relationship matrix for these criteria as represented in Table A7 (Appendix A). The diagonals are filled with 1 as the importance of inflation concerning itself is the same. The table was filled with the data obtained from the questionnaires, and then sent to the experts so that the importance of each criteria factor as it relates to the others could be analyzed. The responses were recorded and Table A7 (Appendix A) was generated to help further calculate the rankings of criteria. A reciprocal rating was assigned when the second criterion was preferred over the previous one.

In the next step, we summed up all the scores and created the Criteria Comparison Matrix [C] using Equation 1.

$$Ci = \frac{Weighted\ factor}{Sum\ of\ weighted\ factor} \quad (1)$$

Furthermore, the Geometric Fuzzy Mean is calculated using Equation 2.

$$G_{ij} = (\prod C_{ij})^{0.2} \quad (2)$$

Where, i= rows (value=1,2,3,4,5) & j= columns (value=1,2,3,4,5)

$$S_{ij} = \sum_{i=1}^{5} G_{ij} \quad (3)$$

Furthermore, considering Equation 4,

$$p(-1)_{ij} = \frac{1}{S_{ij}} \quad (4)$$

where i=1,2,3 and j=1,2,3

Then these P (-1) values are arranged in increasing order and used in further calculation, (See Equation 5).

$$P(-1)_{33} = P(-1)_{11} \quad (5)$$

Now, we have to calculate Fuzzy Weights using the Equation 6.

$$(FW)_{ij} = G_{ij} * P(-1)_{ij} \quad (6)$$

Where, i=1,2,3,4,5 and j=1,2,3,4,5

A table of Fuzzy Weights is constructed for calculating Average Weight Criteria (Mi) and Normalized Weight (Ni).

For Average Weight Criteria (Mi), we use Equation 7.

$$(Mi)_{ij} = \frac{\sum(FW)_{ij}}{3} \quad (7)$$

Here, if i=1, the Average Weight Criteria (Mi) for the first row will be calculated, j will vary from 1 to 3 and we will obtain the first value. The same is applied for other rows and 5 values of Average Weight Criteria (Mi) are calculated.

In the final step, Normalized Weight (Ni) is calculated using Equation 8.

$$(Ni)_{ij} = \frac{(Mi)_{ij}}{\sum_{i=1}^{5}(Mi)_{ij}} \quad (8)$$

Where i=1,2,3,4,5 and j=1

In this way, the five values of Normalized Weight (Ni) were calculated and ranked based on those values. After finding the weight factor, a consistency check was applied to the matrix [C] for cross-verification of the obtained results.

4.2.2 Forecasting (Moving Mean Average)

The initial application of the technique can be found in a study carried out by Holt in 1957 (Holt, 2004). The second part of the analysis involves forecasting using the aforementioned method and, in this case, the past data on interest rates was used to predict interest rates for the future. This was achieved using the technique of Moving Mean Average (MMA) for ten years as shown in Equation 9. These values of mean interest rates are further used to calculate Present Worth (PW).

$$(MMA)i = \frac{I1+I2+I3+I4+I5+I6+I7+I8+I9+I10}{10} \quad (9)$$

Where i= year in which the interest rate is to be predicted and I= interest rates for ten years before the year in which we want to predict the interest rate.

4.2.3 Present Worth Criteria (PWC) using Triangular Fuzzy Numbers (TFN)

The initial application of PWC can be found in a study that applied the technique for the assessment of cash flows (Chiu & Park, 1994) and in the same way, the next phase of the current research will deal with the fuzzy cash flow analysis. This involves taking a variable value for P_0 (X_1, X_2, X_3) and P_1 (X_4, X_5, X_6), and then calculating a Present Worth Value for 2010 - 2019, as shown in the example derivation Equations 10 - 15 below:

$P_o = (X_1, X_2, X_3)$

$P_1 = (X_4, X_5, X_6)$

$R_1 = (r_1, r_2, r_3)$

$P_o = (X_1+(X_2-X_1)\ \alpha, X_3-(X_3-X_2)\ \alpha)$

$P_1 = (X_4+(X_5-X_4)\ \alpha, X_6-(X_6-X_5)\ \alpha)$

$R_1 = (r_1+(r_2- r_1)\ \alpha,\ r_3-(r_3-r_2)\ \alpha)$

$$PW^{l(a)} = \frac{X_1+(X_2-X_1)*\alpha+X_4+(X_5-X_4)*\alpha}{r_1+(r_2-r_1)*\alpha} \quad (10)$$

$$PW^{r(a)} = \frac{X_3-(X_3-X_2)*\alpha+X_6-(X_6-X_5)*\alpha}{r_3-(r_3-r_2)*\alpha} \quad (11)$$

Considering PWA= (PWA_0, PWA_1, PWA_2), where

$$PWA_0 = \frac{(X_1+X_4)}{(1+r_3)} \quad (12)$$

$$PWA_1 = \frac{(X_2+X_5)}{(1+r_2)} \quad (13)$$

$$PWA_2 = \frac{(X_3+X_6)}{(1+r_1)} \quad (14)$$

Therefore, PWA= [$PWA^{I(\alpha)}$, $PWA^{r(\alpha)}$] = [PWA_0+(PWA_1–PWA_0) *α, PWA_2+(PWA_1–PWA_2) *α] (15)

The weighted average method was used to forecast the Lowest, Optimum and Highest NPVs (Net Present Values) in terms of the cash flows P_0 (X_1, X_2, X_3) and P_1(X_4, X_5, X_6). Furthermore, TFM (Triangular Fuzzy Method) was used to find the lowest by incorporating α=0 in the left-hand side of the equation, the optimum by putting α=1 in either side of the equation, and the highest by putting α=0 in the right-hand side of the solution. This will help companies find the lowest, optimum, and highest NPV for their respective cash flows and therefore be able to determine the lowest profit or loss, the optimum profit, and the highest profit for a company's project or investment.

5. Results and discussion

The previous discussion showed how the fuzzy Analytical Hierarchy Process (AHP) can be utilized to quantify the data from the opinions of different experts. This study used the most common approach to create a fuzzy analysis of the different selected criteria by distributing a questionnaire to experts and ranking the criteria based on the AHP analysis performed by the equations described in the methodology. The fuzzy averaged weights (Mi) and normalized weights (Ni) are shown in Table 3.

Table 3
Average weighted criterion and normalized weights

CRI	**Mi**	**Ni**	**Rank**
Inflation	0.055	0.437	1
Political Instability	0.031	0.240	2
Unemployment rate	0.013	0.102	4
GDP rate	0.022	0.175	3
Liquidity rate	0.006	0.045	5
Total	0.127		

As shown in Table 3, C1 criteria inflation, is ranked first. Inflation is the sudden increase in general prices and the economy and hence an increase in the inflation rate has a drastic negative or inverse effect on the value of the interest rate of that year. An increase in the inflation rate causes a decrease in the interest rate. This relationship can be further confirmed by the Fisher Effect as stated by Equation 16.

$$r = i - \pi \tag{16}$$

where, r → real interest rate, i → nominal interest rate, and π → Inflation.

The second ranked criterion is political instability which is indicated by a switch in the government or other regime of the country. Other factors that play an important role in this instability are violent protests, assassinations, suicide bombings, etc. These factors have a history of drastically disturbing the economy of Pakistan, making criterion C2 political instability second in the fuzzy AHP outcome.

Gross Domestic Product (GDP) ranked third since its growth rate is essential to determining the performance of a country GDP and is also considered a measure of how a country's exports and imports balance out, how much the government is investing in something and the government's spending; therefore, an increase in GDP has an increasing effect on the interest rate as well. However, this effect is less than inflation and political instability as GDP is a measure of the performance of the economy, not a direct factor that can create a disturbance in the economy like the other two criteria.

Furthermore, the criterion that ranked fourth is the unemployment rate. This criterion may be a good measure of performance for a government or an economy as more people getting jobs strengthens the economy. However, to consider it to have a direct impact on the fluctuations in interest rates is misleading since both characteristics have no solid effects on each other. Lastly, the liquidity rates help determine whether or not the country can pay off its loans. This data set is crucial for a country like Pakistan which is in great

debt to the U.N (United Nations), the USA (United States of America), and others. Therefore, having a high liquidity rate helps regain economic corridors between different countries, but has very little effect on the state bank interest rates.

Next, we will forecast fuzzy interest rates. Data on fuzzy interests from 2010-2019 were used and were calculated earlier using Fuzzy AHP for the interest rates. A ten-year moving average approach was used as a forecasting method and applied until the year 2025 in the timeline. The results are summarized in Table 4.

Table 4
Ten-year average forecasting on fuzzy interest rates

Year	Minimum	Forecasted values	Middle	Forecasted values	Maximum	Forecasted values
2010	12.31649393		13.5		19.65424577	
2011	11.29547544		12.75		14.99006003	
2012	8.562093556		10		11.88132904	
2013	8.304223864		9.75		11.75639071	
2014	8.295395843		9.5		11.59570423	
2015	6.264827648		7.25		8.964946615	
2016	4.929994857		5.75		7.234855396	
2017	5.103727793		6		7.398007824	
2018	6.592049048		7.7		9.62470587	
2019	10.2326825		11.625		14.1317909	
2020	8.189696448	8.189696448	9.3825	9.3825	10.38483486	11.72320364
2021	7.7770167	7.7770167	8.97075	8.97075	10.79626255	10.79626255
2022	7.425170826	7.425170826	8.592825	8.592825	10.3768828	10.3768828
2023	7.311478553	7.311478553	8.4521075	8.4521075	10.22643818	10.22643818
2024	7.212204021	7.212204021	8.32231825	8.32231825	10.07344292	10.07344292
2025	7.103884839	7.103884839	8.204550075	8.204550075	9.921216791	9.921216791

The Triangular Fuzzy Number method was compatible with our main aims and objectives, and the main reason for the use of triangular fuzzy number was the formulation of an optimal value in the present worth analysis. For the present worth analysis, we needed to calculate the optimal solution with two other maximum and minimum values of profit. Thus, three values were evaluated. As a result, three values for interest rates were calculated and modeled as a triangular fuzzy methodology. In Table 4, the results show fluctuations in the interest rate values. These fluctuations are due to the three most highly ranked criteria as determined by the Analytic Hierarchy Process. The use of fuzzy AHP with normalization methods eradicates vagueness and imprecision from the model. The normalization method used in this procedure transformed qualitative data into quantitative data and then checked for normality and consistency. This normalized section is multiplied by qualitative data from the Analytic Hierarchy Process.

The dual normalization allows values to be set inside a specific range from 0 to 1. This allows for greater accuracy in terms of the percentage effect of each criterion on the interest rate fluctuation.

The next step involves the use of a moving average to forecast the values for future years. A total of 6 forecasts are articulated. These include forecasts from 2020-2025 as shown in Table 4, which depicts the application of the moving average approach. The moving average is applied with the use of the data from the last 10 years, thus a 10-year moving average was calculated. The use of the moving average proved to be a much more accurate and well-interpreted technique. The use of a moving average with a greater n-time period allows for a greater extent of trend smoothing. This ensures a greater degree of accuracy and also takes into account the historical data, thus eliminating vagueness and increasing precision.

One of the main aims of this research is to calculate the Present Worth Analysis (PWA) and for that purpose, forecasting was carried out because we want future interest rates to calculate the Present Worth of any investment. For example, if someone is willing to invest in 2021, he or she can simply obtain the interest rate value from Table A1 in Appendix A and calculate if it is worth investing his or her money. Even if he or she is willing to invest in 2020, our table can be extended within a matter of minutes to obtain an approximated value of the interest rate, applicable in that year. Immense significance is given to the present worth calculations, thus ensuring a greater degree of accuracy. This method proves to be important for local as well as foreign investors. Other major results are shown in Appendix A.

The current research possesses various applications that can be feasible for a country and organization, ranging from a sustainability to an economic point of view. First, the study provides a more practical quantitative approach that has unlimited possibilities to be utilized in similar scenarios, involving various experts and updated secondary resources. Second, the model can work in any setting, i.e., a developing or developed country scenario, providing that the purpose is to assess financial data and incorporate financial forecasting to evaluate the unbalanced interest rates. Third, the study can also be used to attract local and foreign investors by providing them with the latest numbers on the trends involved, thus creating trust in the investing environment. Policymakers and the decision-makers can use this study to develop better fiscal policies to ensure stability in the economic environment and provide better incentives to organizations that are looking to establish their businesses within the country. Also, better policies will always lead to stability in the economic structure, allowing the investors to evaluate their assets before investing, and providing a pathway for better judgment and well-informed decisions.

The study also opens the path for future research incorporating various other MCDM techniques in the context of developing or developed countries. Future studies can also incorporate the classification learner concept by utilizing various coding software to train the data and analyze the model to produce better-forecasted results.

6. Conclusion

This research concludes by proposing and explaining a well-ordered hybrid technique for present-worth analysis. A step-wise approach was used to reach the final solution. This step-wise approach allowed for a detailed interpretation of each phase involved. The initial stages involved a detailed literature review that allowed the development of a knowledge base for our research topic. Both primary and secondary sources were used for data collection. A greater variety of data allowed for greater precision in the results. After successful data collection, the data was utilized in the main methodology part of our research.

This study involved the use of core concepts of the engineering economy and therefore, multiple engineering economy techniques were merged. The first technique involved the use of a fuzzy Analytic Hierarchy Process (AHP) to rank criteria. This is one of the most widely known and utilized multi-criteria decision modeling techniques. A detailed literature review as well as collections of expert opinions allowed us to choose specific criteria for our model. The next approach was the fuzzy triangular method which was used to formulate a three-set conjunction for interest rates. Subsequently, these triangular sets were then used for forecasting using moving averages. Finally, present worth criteria were implemented on the forecasted triangular interest rates. All of these techniques were merged to create the final model. The research focuses on increasing the opportunities for wealth creation and abundance in Pakistan's economy due to the practical use of the proposed model in the study, which ultimately will lead to an uplifting economy and increased welfare in the society.The research primarily focused on Pakistan's fluctuating interest rates related to the State Bank of Pakistan. Moreover, this study could be used to attract local and foreign investors to Pakistan. For any investor, the country's fiscal policy holds significant importance and specifically for local Pakistani investors this research can be used to predict any future interest rate and formulate the value of money accordingly. Thus, a company can review its assets and liabilities in light of future variations. In addition to the State Bank of Pakistan, this research possesses wider promise in providing a baseline for the setup of other commercial banks.

The current research comprises a hybrid combination of techniques to evaluate the fluctuating interest rates in the state bank of Pakistan, a developing country. The study involves the critical collection of data from primary and secondary sources but it also includes limitations that can be addressed in future research. One of the most common limitations that the researchers faced was the unavailability of data because of governmental restrictions. Large efforts were made to access the data sources and many requests were made to connect with experts working in the concerned departments. Second, time constraints were another factor that limited our ability to secure more data sources and responses from the experts. Both of these factors led to the challenge of forecasting financial resources because of the limited resources to keep up with the latest ever-changing trends. Finally, the limitation of the fuzzy AHP method was due to the limited number of responses acquired and also the subjectivity in setting the values for evaluation indexes that could lead to random judgment issues with the results. This issue can be resolved by collecting more responses and also ample amount of time to evaluate the data.

REFERENCES

Abdullah, L., Ong, Z. & Rahim, N., 2021. An intuitionistic fuzzy decision-making for developing cause and effect criteria of subcontractors selection. *International Journal of Computational Intelligence Systems, 14*(1), 991‑1002. Doi; http://dx.doi.org/10.2991/ijcis.d.210222.001

Abel, B., Gray, D., Islam, A. & Bhuiyan, S., 2021. A literature review of the economics of COVID-19. *Journal of Economic Surveys, 35*(4), 1007‑1044. Doi: http://dx.doi.org/10.1111/joes.12423

Ajaz, H., Ahmed, F. & Shehzadi, Y., 2022. Macroeconomic determinants of banks' performance in Pakistan. *International Journal of Experiential Learning & Case Studies, 7*(1), 56‑74. Doi: http://dx.doi.org/10.22555/ijelcs.v7i1.760

Ali, Y., Mehmood, B., Huzaifa, M., Yasir, U., & Ullah Khan, A., 2020a. Development of a new hybrid multi criteria decision-making method for a car selection scenario. *Facta Universitatis, Series: Mechanical Engineering, 18*(3), 357–373. Doi: http://dx.doi.org/10.22190/fume200305031a

Ali, Y., Arif, S., Munir, F., Bilal Azeem, M. & Ullah Khan, A., 2020b. Can India-Pakistan sports promote peace? A MCDM approach. *International Journal of the Analytic Hierarchy Process, 12*(1), 60–81. Doi: http://dx.doi.org/10.13033/ijahp.v11i3.642

Ali, Y., Haroon, M., Abdullah, M. & Khan, A. U., 2019. The best manufacturing procedure for the commercial production of urea, using AHP based TOPSIS. *International Journal of the Analytic Hierarchy Process, 11*(3), 313–330. Doi: http://dx.doi.org/10.13033/ijahp.v11i3.636

Alamoudi, M. H. & Bafail, O. A., 2022. BWM—RAPS Approach for Evaluating and Ranking Banking Sector Companies Based on Their Financial Indicators in the Saudi Stock Market. *Journal of Risk and Financial Management, 15*(10), 1–20. Doi: http://dx.doi.org/10.3390/jrfm15100467

Angelina, S. & Nugraha, N. M., 2020. Effects of monetary policy on inflation and national economy based on analysis of bank Indonesia annual report. *Technium Social Sciences Journal, 10*(1), 423‑435. Doi: http://dx.doi.org/10.47577/tssj.v10i1.1300

Berliner, B. & Buehlmann, N., 1993 [2014]. A generalization of the fuzzy zooming of cash flows. *ASTIN Bulletin: The Journal of the IAA*, *23*(2), 431–456. Rome: 3rd AFIR International Colloquium.

Bollos, M.-I., Bradea, I.-A. & Delcia, C., 2019. The development of a fuzzy logic system in a stochastic environment with normal distribution variables for cash flow deficit detection in corporate loan policy. *Symmetry, 11*, 1–18. Doi: http://dx.doi.org/10.3390/sym11040548

Boltürk, E. & Kahraman, C., 2022. Interval-valued and circular intuitionistic fuzzy present worth analyses. *Informatica, 33*(4), 693–711. Doi: http://dx.doi.org/10.15388/22-infor478

Boltürk, E. & Seker, S., 2022. *Present worth analysis using spherical fuzzy sets*. Istanbul: Springer.

Castillo , O., Castro, J. R. & Melin, P., 2022. Interval type-3 fuzzy aggregation of neural networks for multiple time series prediction: the case of financial forecasting. *Axioms, 11*(6), 1–13. Doi: http://dx.doi.org/10.3390/axioms11060251

CEIC, n.d. Pakistan PK: Deposit Takers: Liquidity: Liquid Assets to Total Assets. https://www.ceicdata.com/en/pakistan/financial-soundness-indicators-annual/pk-deposit-takers-liquidity-liquid-assets-to-total-assets

Chiu, C.-Y. & Park, C. S., 1994. Fuzzy cash flow analysis using present worth criterion. *The Engineering Economist, 39*(2), 113. Doi: http://dx.doi.org/10.1080/00137919408903117

countryeconomy.com, n.d. Pakistan - Key rates. https://countryeconomy.com/key-rates/pakistan

Dimitrovski, A. & Matos, M., 2008. Fuzzy present worth analysis with correlated and uncorrelated cash flows. In Cengiz Kahraman (Ed.) *Fuzzy engineering economics with applications* (pp.10–41). Washington: Scientific Publishing Services. Doi: http://dx.doi.org/10.1007/978-3-540-70810-0_2

Hamal, S. & Senvar, O., 2022. A novel integrated AHP and MULTIMOORA method with interval-valued spherical fuzzy sets and single-valued spherical fuzzy sets to prioritize financial ratios for financial accounting fraud detection. *Journal of Intelligent & Fuzzy Systems, 42*(1), 337–364. Doi: http://dx.doi.org/10.3233/jifs-219195

Hasan, S., 2016, February 8. Expanding fleet: Amid PIA crisis, private airlines start inducting 9 jets. *The Express Tribune*. https://tribune.com.pk/story/1042826/expanding-fleet-amid-pia-crisis-private-airlines-start-inducting-9-jets

Holt, C. C., 2004. Forecasting seasonals and trends by exponentially weighted moving averages. *International Journal of Forecasting, 20*(1), 5–10. Doi: http://dx.doi.org/10.1016/j.ijforecast.2003.09.015

Insight Securities, 2022. *Pakistan Economy*. Karachi: Insight Research.

Khan, A. U. & Ali, Y., 2020. Analytical hierarchy process (AHP) and analytic network process methods and their applications: a twenty year review from 2000-2019. *International Journal of the Analytic Hierarchy Process, 12*(3), 369-459. Doi: http://dx.doi.org/10.13033/ijahp.v12i3.822

Krustev, G., 2019. The natural rate of interest and the financial cycle. *Journal of Economic Behavior & Organization, 162*(1), 193-210. Doi: http://dx.doi.org/10.1016/j.jebo.2018.12.024

Mahtani, U. S. & Garg, C. P., 2018. An analysis of key factors of financial distress in airline companies in India using fuzzy AHP framework. *Transportation Research Part A: Policy and Practice, 117*(1), 87-102. Doi: http://dx.doi.org/10.1016/j.tra.2018.08.016

Malik, F. J. & Nishat, M., 2017. Real interest rate volatility in the Pakistani economy: A regime switching approach. *Business Review*, 22–32. Doi: http://dx.doi.org/10.54784/1990-6587.1041

Marques, A. I., Garcia, V. & Sanchez, J. S., 2020. Ranking-based MCDM models in financial management applications: analysis and emerging challenges. *Progress in Artificial Intelligence, 9*(1), 171–193. Doi: http://dx.doi.org/10.1007/s13748-020-00207-1

Martínez-Peláez, R. et al., 2023. Role of digital transformation for achieving sustainability: mediated role of stakeholders, key capabilities, and technology. *Sustainability, 15*(14), 1-27. Doi: http://dx.doi.org/10.3390/su151411221

Mazhar, U. & Rehman, F., 2021. Monetary policy in a developing country: A case of Pakistan. *Asian Journal of Management Cases*, *18*(2), 144-155. Doi: http://dx.doi.org/10.1177/09728201211028719

Mohammadian, M. & Kingham, M., 2004. An adaptive hierarchical fuzzy logic system for modelling of financial systems. *Intelligent Systems in Accounting, Finance & Management: International Journal, 12*(1), 61–82. Doi: http://dx.doi.org/10.1002/isaf.241

Nazim, M., Mohammad, C. W. & Sadiq, M., 2022. A comparison between fuzzy AHP and fuzzy TOPSIS methods to software requirements selection. *Alexandria Engineering Journal, 61*(12), 10851–10870. Doi: http://dx.doi.org/10.1016/j.aej.2022.04.005

Nguyen, T. A. V., Tucek, D. & Pham, N. T., 2023. Indicators for TQM 4.0 model: Delphi method and analytic hierarchy process (AHP) analysis. *Total Quality Management & Business Excellence, 34*(1), 220–234. Doi: http://dx.doi.org/10.1080/14783363.2022.2039062

Rafiq, M. Z., Jun, J. C., Naseem, S. & Mohsin, M., 2019. Impact of market risk, interest rate, exchange rate on banks stock return: evidence from listed banks of Pakistan. *Amazonia Investiga*, *8*(21), 667-673.

Saaty, T. L. & Vargas, L. G., 1979. Estimating technological coefficients by the analytic hierarchy process. *Socio-Economic Planning Sciences, 13*(6), 333–336. Doi: http://dx.doi.org/10.1016/0038-0121(79)90015-6

Samal, S. & Dash, R., 2022. A novel MCDM ensemble approach of designing an ELM based predictor for stock index price forecasting. *Intelligent Decision Technologies, 16*(2), 387–406. Doi: http://dx.doi.org/10.3233/idt-210152

Sarfaraz, A. H. et al., 2023. A novel hierarchical fuzzy inference system for supplier selection and performance improvement in the oil & gas industry. *Journal of Decision Systems, 32*(2), 356–383. Doi: http://dx.doi.org/10.1080/12460125.2022.2090065

Shen, K.-Y., Hu, S.-K. & Tzeng, G.-H., 2017. Financial modeling and improvement planning for the life insurance industry by using a rough knowledge based hybrid MCDM model. *Information Sciences, 375*(1), 296–313. Doi: http://dx.doi.org/10.1016/j.ins.2016.09.055

State Bank of Pakistan, various dates. Working papers-State Bank of Pakistan. http://www.sbp.org.pk/publications/wpapers/index2.asp

Statista, n.d. Inflation rate in Pakistan: Inflation rate from 1987-2028. https://www.statista.com/statistics/383760/inflation-rate-in-pakistan/

Song, Y. & Peng, Y., 2019. A MCDM-based evaluation approach for imbalanced classification methods in financial risk prediction. *IEEE Access, 7*(1), 84897–84906. Doi: http://dx.doi.org/10.1109/access.2019.2924923

Taliento, M., 2019. Corporate valuation: Looking beyond the forecast period through new "fuzzy lenses. *IEEE Transactions on Engineering Management, 68*(2), 467–482. Doi: http://dx.doi.org/10.1109/tem.2019.2904955

Ul Mustafa, A. R., Abro, A. A., Hussain, T. & Ali, S. R., 2021. Populism, seigniorage and inequality dilemma in perspective of Pakistan. *Academy of Accounting and Financial Studies Journal, 25*(S4), 1–14.

Wang, Q. & Jiang, F., 2019. Integrating linear and nonlinear forecasting techniques based on grey theory and artificial intelligence to forecast shale gas monthly production in Pennsylvania and Texas of the United States. *Energy, 178*(1), 781-803. Doi: http://dx.doi.org/10.1016/j.energy.2019.04.115

World Bank, n.d.a Unemployment, total (% of total labor force) (modeled ILO estimate) - Pakistan. https://data.worldbank.org/indicator/SL.UEM.TOTL.ZS?locations=PK

World Bank, n.d.b GDP growth (annual %) - Pakistan. https://data.worldbank.org/indicator/NY.GDP.MKTP.KD.ZG?locations=PK

Appendix A

Procedure:

An example of the entire procedure is shown below in the form of tables from Excel for the year 2015.

Table A1
Relationship of each criterion with interest rate

Relationship with interest rate	**2015 Criterion**	**2015 Rates**	
Inverse	Inflation	4.53	20
Direct	GDP growth rate	4.731	12
Direct	Liquidity	46.937	100
Direct	Political instability	6	10
Direct	Unemployment rate	3.57	4

Table A2
Relative ratios which affected the interest rates

Relative ratios which affected the interest rates in an increasing manner	**Normalized ratios**
Inflation 0.7735	0.247154607
GDP (Gross Domestic Product) Growth Rate 0.39425	0.12597376
Liquidity 0.46937	0.149976674
Political instability 0.6	0.191716566
Unemployment rate 0.8925	0.285178392
Sum = 3.12962	1

Table A3
Final Relative ratios that affect the interest rate in 2015

Normalized ranks from the questionnaire	**Final relative ratios that affect the interest rate in 2015**
0.437	0.108006563
0.175	0.022045408
0.046	0.006898927
0.24	0.046011976
0.102	0.029088196
1	0.21205107

Table A4
Final ranks of criterion using normalized ratios

Final normalized ratios that affect the interest rate in 2015	**Ranks**
Inflation 0.509342221	1
GDP Growth Rate 0.10396273	4
Liquidity 0.032534271	5
Political instability 0.216985351	2
Unemployment rate 0.137175427	3
The sum of the values = 1	

Table A5
Triangular fuzzy values of interest rates in 2015

Actual Value		
6.260397	7.25	8.958606
0.062604	0.0725	0.089586

Similarly, Table A6 depicts the calculation of the fuzzy Present Worth for a given cash flow.

Table A6
Net present value of a dummy cash flow for 2015

	Cash flow P0				**Cash flow P1**	
-110	-100	-90		100	110	130
PWA0			**PWA1**			**PWA2**
-9.1778			9.324009			37.64338
PWAL		**PWA_Op1**		**PWA_Op2**		**PWAH**
-9.1778		9.324009		9.324009		37.64338

Here, PWAL is the lowest possible present worth or highest loss, while PWA_Op1 and PWA_Op2 are the optimum profits and PWAH is the highest present worth a company will get if they invest in Pakistan in 2015 with the above-mentioned cash flows P0 and P1.

Table A7
Relationship matrix

Criteria	**Inflation**	**GDP (Gross Domestic Product)**	**Liquidity**	**Political Instability**	**Unemployment Rates**
Inflation	1,1,1	1,2,4	2,4,6	2,4,6	4,6,8
Political instability	1/4,1/2,1	1,1,1	1,2,4	1,2,4	2,4,6
Liquidity	1/6,1/4,1/2	1/4,1/2,1	1,1,1	1/6,1/4,1/2	2,4,6
GDP rate	1/6,1/4,1/2	1/4,1/2,1	2,4,6	1,1,1	4,6,8
Unemployment rates	1/8,1/6,1/4	1/6,1/4,1/2	1/6,1/4,1/2	1/8,1/6,1/4	1,1,1