ANALYTIC HIERARCHY PROCESS BASED ON THE MAGNITUDE OF Z-NUMBERS

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ABSTRACT

The Analytic Hierarchy Process (AHP) is a powerful multi-criteria and multi-alternative decision-making model, which assists decision-makers in giving preferences using pairwise comparison matrices. The development of the AHP using fuzzy numbers has received attention from many researchers due to the ability of fuzzy numbers to handle vagueness and uncertainty. The integration of the AHP with fuzzy Z-numbers has improved the model since the reliability of the decision-makers is considered, in which the judgment is followed by a degree of certainty or sureness. Most of the existing decision-making models based on Z-numbers transform the Z-numbers into regular fuzzy numbers by integrating the reliability parts into the restriction parts, causing a significant loss of information. Hence, this study develops the AHP based on the magnitude of Z-

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numbers, which is used to represent the criteria weights. A numerical example of criteria ranking for the prioritization of public services for digitalization is implemented to illustrate the proposed AHP model.

Keywords: AHP; magnitude; Z-numbers; criteria ranking

1. Introduction

Humans tend to describe almost everything with natural language, and the description is mainly based on cognitive and psychological factors. The decision-making process is related to the cognitive process, in which human thinking is involved. In the real world, human preferences are not well-defined (Perote-Peña & Piggins, 2007). According to Aliev et al. (2021), human preferences in the decision-making process are imprecise due to the complexity of alternatives, imperfect information, and psychological biases.

In the early development of decision-making methods, crisp numbers were used to describe decision-makers' preferences. However, due to the lack of information, the use of crisp numbers has led to uncertainty (Aliev, 2013). The implementation of fuzzy sets introduced by Zadeh (1965) can handle the uncertainty of preference degrees.

Accordingly, many multi-criteria decision-making (MCDM) methods have been developed to help decision-makers select the best alternatives when there are various attributes that need to be considered. The Analytical Hierarchy Process (AHP) is one of the most powerful of these methods. The AHP was proposed by Saaty (1980) and uses a pairwise comparison matrix to obtain the evaluation of decision makers.

The AHP has been studied extensively due to the fact that it is simple, easy to use, and flexible (Emrouznejad & Ho, 2017). It has also been implemented to solve decision-making problems with many criteria in various fields such as education, management, engineering, manufacturing, and sports (Vaidya & Kumar, 2006).

In 2011, Zadeh introduced the concept of Z-numbers to deal with partially reliable information. Z-numbers are composed of restriction and reliability components; the reliability component describes how certain the preference on the restriction component is made. According to Abdullahi et al. (2020), Z-numbers are the generalization of real, interval, and fuzzy numbers. Moreover, Z-numbers are very powerful in describing decision-making information due to their capability of modeling the real-world.

In this article, the strength of Z-numbers is adopted in developing the AHP based on the magnitude of Z-numbers. Since the magnitude of fuzzy numbers exhibits visual and natural meaning (Abbasbandy & Hajjari, 2009), the magnitude of Z-numbers is used to process the decision information instead of converting Z-numbers into regular fuzzy numbers that lead to information loss (Abdullahi et al., 2020; Gardashova, 2019; Shen & Wang, 2018).

2. Literature review

The AHP is an additive weighting method described in the pairwise comparison matrix. The AHP was first proposed by Saaty (1980) and was further extended into fuzzy AHP by many researchers due to its simplicity and flexibility. The fuzzy AHP generally replaces the crisp numbers used in the pairwise comparison matrix with fuzzy numbers to handle imprecision in the evaluation of the decision-makers.

Subsequently, van Laarhoven and Pedrycz (1983) extended Saaty's (1980) AHP by using a fuzzy logarithmic least square method to process triangular fuzzy numbers in order to obtain the triangular fuzzy weights. Meanwhile, Ruoning and Xiaoyan (1992) used interval numbers to develop the AHP in a fuzzy environment. The extent analysis method was also used in the fuzzy AHP by Chang (1996). However, the extent analysis was unable to estimate the exact weights from the fuzzy comparison matrices (Wang et al., 2008).

Furthermore, Leung and Cao (2000) defined the consistency of the fuzzy AHP. The fuzzy least-square priority method was also integrated with the AHP by Xu (2000), which produced an analytic expression for the criteria weights. In the following year, Buckley et al. (2001) proposed a direct fuzzification of Saaty’s method to obtain the fuzzy weights. Additionally, Wang et al. (2006) modified van Laarhoven and Pedrycz's (1983) method using a constrained non-linear optimization model, which can directly derive the fuzzy weights for fuzzy pairwise comparison matrices.

Azadeh et al. (2013) extended the AHP based on Z-numbers to solve the selection of private self-financing technical institutions; however, in their proposed model, the Z-numbers were converted into regular fuzzy numbers by integrating the second component into the first component using Kang et al.'s (2012) transformation method. The transformation of Z-numbers into regular fuzzy numbers causes a significant loss of information (Abdullahi et al., 2020; Gardashova, 2019; Shen & Wang, 2018). During the transformation, the reliability component of the Z-numbers was converted into a crisp value, which was added to the restriction component as a weight. The weighted restriction component was then converted into regular fuzzy numbers, dissipating some information in the Z-numbers since the decision information was not well preserved in the form of paired fuzzy numbers.

Meanwhile, a novel AHP based on the direct calculation of Z-numbers was proposed by Zeinalova (2018). Instead of converting Z-numbers into regular fuzzy numbers, the direct calculation using the arithmetic operations of Z-numbers proposed by Aliev et al. (2015) was used. Although this method could avoid the issue of information loss, the method may, however, lead to high computational complexity caused by the extensive use of linear programming to solve simple problems (Abdullahi et al., 2020).

In the magnitude of Z-numbers derived by Farzam et al. (2021), the magnitudes of the restriction and reliability components of the Z-numbers were combined using a convex compound. Since the magnitude of fuzzy numbers is visual and natural (Abbasbandy & Hajjari, 2009), it is acceptable to develop the AHP based on the magnitude of Z-numbers to preserve the decision information and model an efficient decision-making method.

3. Preliminaries

In general, a fuzzy number is an extension of Zadeh's fuzzy set, which entails a fuzzy subset of the real line, particularly whose maximum membership degrees are clustered around the average value. The prominent shapes of fuzzy numbers are triangular and trapezoidal. Since the triangular fuzzy number is the simplest form of a fuzzy number (Voskoglou, 2019), its application in developing the decision-making model in this article is, therefore, easier. The triangular fuzzy number is defined as follows:

Definition 1 (Zhang et al., 2014)**:**
Let $\tilde{\alpha} = (a_1, a_2, a_3)$ be a triangular fuzzy number. As such, its membership degree is characterized by

$$\mu_{\tilde{\alpha}}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1} & , \ x \in [a_1, a_2] \\ \dfrac{a_3 - x}{a_3 - a_2} & , \ x \in [a_2, a_3] \\ 0 & , \ \text{elsewhere.} \end{cases} \tag{1}$$

Definition 2 (Zhang et al., 2014)**:**
Suppose that $\tilde{\alpha} = (a_1, a_2, a_3)$ and $\tilde{\beta} = (b_1, b_2, b_3)$ are triangular fuzzy numbers and λ is a scalar. Accordingly,

(i) $\tilde{\alpha} + \tilde{\beta} = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$
(ii) $\tilde{\alpha} \times \tilde{\beta} = (a_1 b_1, a_2 b_2, a_3 b_3)$
(iii) $\lambda \tilde{\alpha} = (\lambda a_1, \lambda a_2, \lambda a_3)$
(iv) $\tilde{\alpha}^{-1} = \left(\dfrac{1}{a_3}, \dfrac{1}{a_2}, \dfrac{1}{a_1} \right)$

Zadeh (2011) extended the classical fuzzy number into the Z-number, which consists of both the restriction and reliability components.

Definition 3 (Zadeh, 2011):
A Z-number, $Z = (\tilde{A}, \tilde{R})$ consists of two components. The first component, $\tilde{A}$ represents the restriction on the value that a variable can take. The second component, $\tilde{R}$ represents the degree of reliability or certainty of the first component.

For simplicity, both components of the Z-numbers are represented by triangular fuzzy numbers, as shown in Figure 1.

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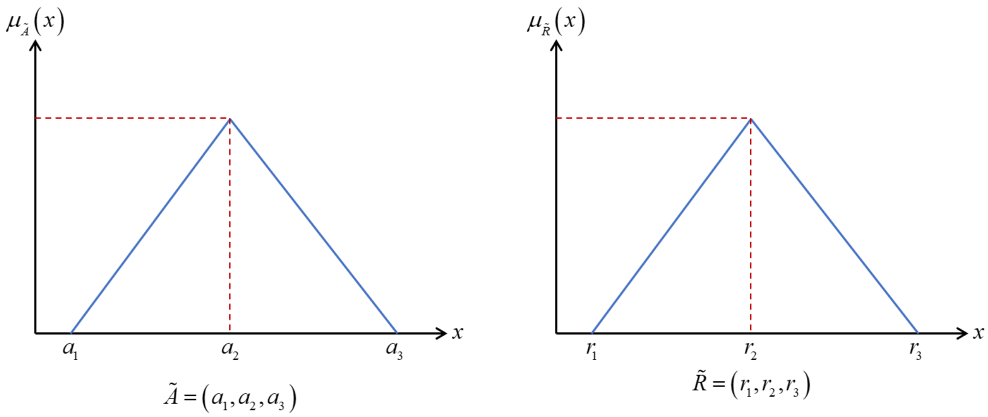

Figure 1 Z-number, $Z=(\tilde{A},\tilde{R})$

Recently, Farzam et al. (2021) proposed the magnitude of Z-numbers by combining the magnitude of the first and second components using the concept of a convex compound.

Definition 4 (Farzam et al., 2021):
Let $\tilde{A}=(a_1,a_2,a_3,a_4)$ and $\tilde{R}=(r_1,r_2,r_3,r_4)$ be the restriction and reliability components of a Z-number, respectively. In this regard, the magnitude of $\tilde{A}$ and $\tilde{R}$ are given by

$$Mag(\tilde{A})=\frac{1}{12}(a_1+5a_2+5a_3+a_4) \tag{2}$$

and

$$Mag(\tilde{R})=\frac{1}{12}(r_1+5r_2+5r_3+r_4), \tag{3}$$

respectively. Hence, the magnitude of the Z-number, $Z=(\tilde{A},\tilde{R})$ is given by

$$Mag(Z)=\lambda Mag(\tilde{A})+(1-\lambda)Mag(\tilde{R}) \tag{4}$$

where $\lambda\in[0.5,1]$ to highlight that the first component is more important in representing the Z-number.

Furthermore, Farzam et al. (2021) formed some rules to rank the Z-numbers based on the magnitude values.

Definition 5 (Farzam et al., 2021):
Let $Z_1=(\tilde{A}_1,\tilde{R}_1)$ and $Z_2=(\tilde{A}_2,\tilde{R}_2)$ be two Z-numbers having the magnitude defined in Equation 4. Accordingly,

(i) $Z_1 \sim Z_2$ if $\lambda\left[Mag(\tilde{A}_1)-Mag(\tilde{A}_2)\right]=(1-\lambda)\left[Mag(\tilde{R}_1)-Mag(\tilde{R}_2)\right]$

(ii) $Z_1 \prec Z_2$ if $\lambda\left[Mag\left(\tilde{A}_1\right) - Mag\left(\tilde{A}_2\right)\right] < (1-\lambda)\left[Mag\left(\tilde{R}_1\right) - Mag\left(\tilde{R}_2\right)\right]$

(iii) $Z_1 \succ Z_2$ if $\lambda\left[Mag\left(\tilde{A}_1\right) - Mag\left(\tilde{A}_2\right)\right] > (1-\lambda)\left[Mag\left(\tilde{R}_1\right) - Mag\left(\tilde{R}_2\right)\right]$

4. Proposed AHP based on the magnitude of Z-numbers

In this section, the AHP based on the Z-numbers is proposed. Steps 1 to 3 are contingent on the methodology from Buckley's fuzzy AHP (Buckley, 1985), but were conducted separately on the restriction and reliability components. Step 4 converts the fuzzy weights representing the restriction and reliability components into magnitude values before combining them in Step 5. The combined weight is further normalized in Step 6 before being ranked. The detailed steps for the proposed Z-number-based AHP are as follows:

Step 1: Construct the pairwise comparison matrices to represent the decision maker's preferences on the restriction and reliability of each criterion. While $\tilde{A}_{ij}$ denotes the degree to which the *i*-th criterion is preferred to the *j*-th criterion, $\tilde{R}_{ij}$ represents the degree of reliability when $\tilde{A}_{ij}$ is determined. $\tilde{A}_{ij}$ and $\tilde{R}_{ij}$ are represented by triangular fuzzy numbers.

Table 1
Pairwise comparisons for the restriction of the criteria

	Criterion 1	Criterion 2		Criterion *n*
Criterion 1	$\tilde{A}_{11}$	$\tilde{A}_{12}$	$\cdots$	$\tilde{A}_{1n}$
Criterion 2	$\tilde{A}_{21}$	$\tilde{A}_{22}$	$\cdots$	$\tilde{A}_{2n}$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
Criterion *n*	$\tilde{A}_{n1}$	$\tilde{A}_{n2}$	$\cdots$	$\tilde{A}_{nn}$

Table 2
Pairwise comparisons for the reliability of the restriction of the criteria

	Criterion 1	Criterion 2		Criterion *n*
Criterion 1	$\tilde{R}_{11}$	$\tilde{R}_{12}$	$\cdots$	$\tilde{R}_{1n}$
Criterion 2	$\tilde{R}_{21}$	$\tilde{R}_{22}$	$\cdots$	$\tilde{R}_{2n}$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
Criterion *n*	$\tilde{R}_{n1}$	$\tilde{R}_{n2}$	$\cdots$	$\tilde{R}_{nn}$

Step 2: The pairwise comparison matrices are then aggregated using the following geometric mean:

$$G\left(\tilde{\alpha}_i\right) = \left(\prod_{j=1}^{n} \tilde{\alpha}_{ij}\right)^{\frac{1}{n}} \tag{5}$$

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where $\tilde{\alpha}$ denotes any arbitrary triangular fuzzy number that satisfies Definition 2. Note that the aggregated mean is a triangular fuzzy number.

Step 3: The fuzzy weights representing the restriction and reliability components are calculated. The aggregated triangular fuzzy numbers are summed using the formula below.

$$S_{G(\tilde{\alpha})} = \sum_{i=1}^{n} G\left(\tilde{\alpha}_i\right) \tag{6}$$

For each criterion $i = 1, 2, \ldots, n$, the fuzzy weights are calculated using the formula below.

$$N_{\tilde{\alpha}_i} = S_{G(\tilde{\alpha})}^{-1} \times G\left(\tilde{\alpha}_i\right) \tag{7}$$

Step 4: Calculate the magnitude of the restriction and reliability components, as follows:

$$Mag\left(N_{\tilde{\alpha}_i}\right) = \frac{1}{12}\left[n_1 + 10n_2 + n_3\right] \tag{8}$$

such that $N_{\tilde{\alpha}_i} = \left(n_1, n_2, n_3\right)$. This formula is obtained from Definition 4, in which the trapezoidal fuzzy numbers are assumed as triangular fuzzy numbers.

Step 5: Calculate the weight of each criterion by combining the magnitudes of the restriction and reliability components using the convex compound, as follows:

$$W\left(C_i\right) = \lambda Mag\left(N_{\tilde{A}_i}\right) + \left(1-\lambda\right) Mag\left(N_{\tilde{R}_i}\right) \tag{9}$$

Step 6: The weight is finally normalized using the following formula:

$$\tilde{W}\left(C_i\right) = \frac{W\left(C_i\right)}{\sum_{i}^{n} W\left(C_i\right)} \tag{10}$$

such that $\sum_{i=1}^{n} \tilde{W}\left(C_i\right) = 1$.

Step 7: The criteria are ranked based on the normalized weights.

The above steps of the proposed AHP model based on the magnitude of Z-numbers can be summarized as shown in Figure 2.

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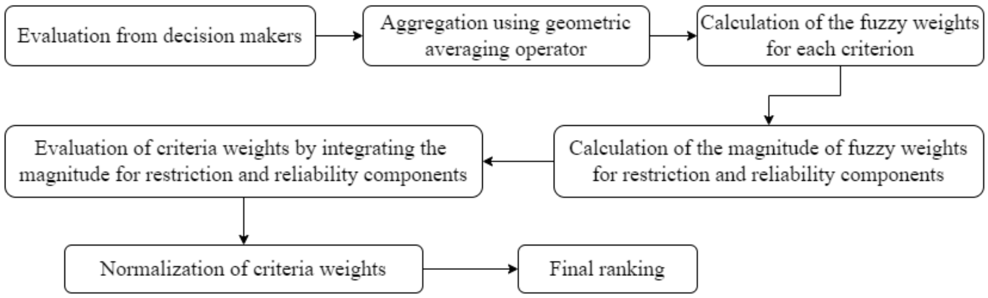

Figure 2 Proposed AHP based on the magnitude of Z-numbers

5. Criteria ranking for the prioritization of public services

The case study from Sergi and Sari (2021) regarding the ranking of criteria to prioritize public services for digitalization was adopted to illustrate the proposed AHP based on the magnitude of Z-numbers. Figure 3 below summarizes the goal and criteria considered in the decision-making problem.

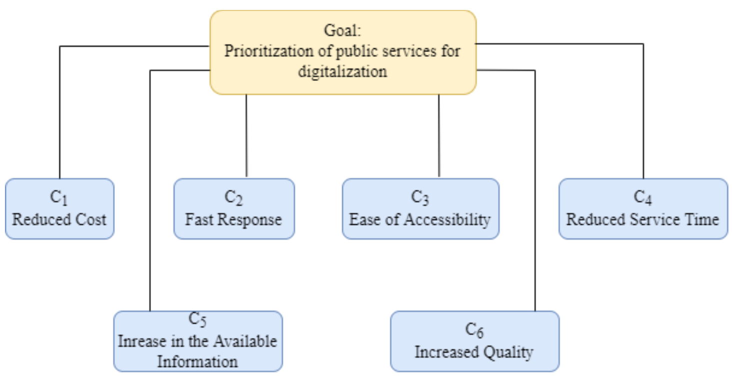

Figure 3 Goal and criteria of the decision-making problem (Sergi & Sari, 2021)

Step 1: The decision maker evaluates the criteria using the pairwise comparison matrix. For the model to be able to handle the partially reliable information, two pairwise comparison matrices are constructed, one each for the restriction and reliability of the decision maker's preferences as shown in Tables 3 and 4, respectively.

Table 3
Pairwise comparisons for the restriction of the criteria

	C_1	C_2	C_3	C_4	C_5	C_6
C_1	EI	RWI	RMI	RMI	GI	WI
C_2	WI	EI	RWI	RWI	GI	MI
C_3	MI	WI	EI	WI	AI	MI
C_4	MI	WI	RWI	EI	GI	MI
C_5	RGI	RGI	RAI	RGI	EI	RWI
C_6	RWI	RMI	RMI	RMI	WI	EI

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Table 4
Pairwise comparisons for the reliability of the criteria

	C_1	C_2	C_3	C_4	C_5	C_6
C_1	AR	VWR	FR	FR	VHR	VHR
C_2	VHR	AR	VWR	VWR	VHR	FR
C_3	FR	VHR	AR	VHR	SR	FR
C_4	FR	VHR	VWR	AR	VHR	FR
C_5	VWR	VWR	SU	VWR	AR	VWR
C_6	VWR	FR	FR	FR	VHR	AR

The decision maker's opinion in natural language is transformed into Z-numbers, in which the restriction and reliability matrices are converted into triangular fuzzy numbers using the linguistic values as shown in Tables 5 and 6, respectively.

Table 5
Linguistic values for the restriction matrix (Sergi & Sari, 2021)

Linguistic Term	**Triangular Fuzzy Numbers**
Equally important (EI)	(1,1,1)
Weakly important (WI)	(1,3,5)
Moderately important (MI)	(3,5,7)
Greatly important (GI)	(5,7,9)
Absolutely important (AI)	(7,9,9)
Reciprocal weakly important (RWI)	(1/5,1/3,1)
Reciprocal moderately important (RMI)	(1/7,1/5,1/3)
Reciprocal greatly important (RGI)	(1/9,1/7,1/5)
Reciprocal absolutely important (RAI)	(1/9,1/9,1/7)

Table 6
Linguistic values for the reliability matrix (Sergi & Sari, 2021)

Linguistic Terms	**Triangular Fuzzy Numbers**
Absolutely reliable (AR)	(1.0,1.0,1.0)
Strongly reliable (SR)	(0.7,0.8,0.9)
Very highly reliable (VHR)	(0.6,0.7,0.8)
Highly reliable (HR)	(0.5,0.6,0.7)
Fairly reliable (FR)	(0.4,0.5,0.6)
Weakly reliable (WR)	(0.3,0.4,0.5)
Very weakly reliable (VWR)	(0.2,0.3,0.4)
Strongly unreliable (SU)	(0.1,0.2,0.3)
Absolutely unreliable (AU)	(0.0,0.1,0.2)

Step 2: The pairwise comparison matrices for the restriction and reliability components are then aggregated using Equation 5. Table 7 below presents the aggregated triangular fuzzy numbers representing the restriction and reliability components for each criterion.

Table 7
Aggregated pairwise comparison matrix

Criterion	Restriction Part	Reliability Part
C_1	(0.523,0.809,1.308)	(0.508,0.605,0.698)
C_2	(0.918,1.506,2.608)	(0.485,0.583,0.677)
C_3	(1.995,3.557,4.718)	(0.586,0.679,0.769)
C_4	(1.442,2.365,3.608)	(0.508,0.605,0.698)
C_5	(0.177,0.218,0.323)	(0.305,0.415,0.515)
C_6	(0.289,0.447,0.755)	(0.475,0.572,0.665)

Step 3: The aggregated triangular fuzzy numbers for the restriction and reliability parts from Table 7 are then summed. Hence, its inverse is calculated using Definition 2.

Table 8
Summation of aggregated pairwise comparison matrix and its inverse

	Restriction Part	Reliability Part
Summation	(5.344,8.902,13.32)	(2.868,3.459,4.021)
Inverse	(0.075,0.112,0.187)	(0.249,0.289,0.349)

Subsequently, the fuzzy weights are calculated using Equation 7. The fuzzy weights obtained are shown in Table 9.

Table 9
Fuzzy weights for all criteria

Criterion	Restriction Part	Reliability Part
C_1	(0.039,0.091,0.245)	(0.126,0.175,0.243)
C_2	(0.069,0.169,0.488)	(0.121,0.169,0.236)
C_3	(0.150,0.400,0.883)	(0.146,0.196,0.268)
C_4	(0.108,0.266,0.675)	(0.126,0.175,0.243)
C_5	(0.013,0.025,0.061)	(0.076,0.120,0.180)
C_6	(0.022,0.050,0.141)	(0.118,0.165,0.232)

Step 4: The magnitude of each of the restriction and reliability parts of Z-numbers is calculated using Equation 8. The magnitudes are presented in Table 10.

Table 10
Magnitude of triangular fuzzy numbers

Criterion	Restriction Part	Reliability Part
C_1	0.0994	0.1766
C_2	0.1874	0.1701
C_3	0.4190	0.1981
C_4	0.2867	0.1766
C_5	0.0266	0.1213
C_6	0.0554	0.1670

Step 5: The weight of each criterion is then determined by calculating the magnitude of the Z-number representing each criterion, in which the magnitudes of the restriction and reliability components are combined using Equation 9. Table 11 below presents the weights of the criteria using several λ values.

Table 11
Criteria weights

Criterion	λ					
	0.5	0.6	0.7	0.8	0.9	1.0
C_1	0.1380	0.1302	0.1225	0.1148	0.1071	0.0994
C_2	0.1788	0.1805	0.1822	0.1839	0.1857	0.1874
C_3	0.3085	0.3306	0.3527	0.3748	0.3969	0.4190
C_4	0.2316	0.2426	0.2536	0.2646	0.2757	0.2867
C_5	0.0740	0.0645	0.0550	0.0455	0.0361	0.0266
C_6	0.1112	0.1001	0.0889	0.0777	0.0666	0.0554

Step 6: Using Equation 10, the criteria weights from Table 11 are subsequently normalized, as shown in Table 12.

Table 12
Normalized criteria weights

Criterion	λ					
	0.5	0.6	0.7	0.8	0.9	1.0
C_1	0.1324	0.1242	0.1161	0.1082	0.1003	0.0925
C_2	0.1716	0.1721	0.1727	0.1733	0.1739	0.1744
C_3	0.2961	0.3153	0.3343	0.3531	0.3716	0.3900
C_4	0.2223	0.2314	0.2404	0.2493	0.2581	0.2668
C_5	0.0710	0.0615	0.0521	0.0429	0.0338	0.0247
C_6	0.1067	0.0954	0.0843	0.0732	0.0624	0.0516

Step 7: Finally, the criteria are ranked based on the normalized weights. For all values of λ, the criteria are ranked as $C_3 \succ C_4 \succ C_2 \succ C_1 \succ C_6 \succ C_5$.

6. Discussion

Table 13 compares the ranking of the criteria obtained using the proposed model with the ranking obtained by Sergi and Sari (2021). The parameter $\lambda = 0.5$ was used to highlight equal roles of the restriction and reliability components in representing the Z-numbers. In fact, the proposed method validates the criteria ranking using multiple λ values, satisfying the fact that the Z-numbers are majorly represented by the restriction component.

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Table 13
Comparison of criteria ranking with the existing method

Criterion	Proposed ($\lambda = 0.5$)		Proposed ($\lambda = 1.0$)		Sergi and Sari (2021)	
	Weight	Ranking	Weight	Ranking	Weight	Ranking
C_1	0.1324	4	0.0925	4	0.094	4
C_2	0.1716	3	0.1744	3	0.172	3
C_3	0.2961	1	0.3900	1	0.399	1
C_4	0.2223	2	0.2668	2	0.264	2
C_5	0.0710	6	0.0247	6	0.019	6
C_6	0.1067	5	0.0516	5	0.052	5

The proposed model produced the same ranking ($C_3 \succ C_4 \succ C_2 \succ C_1 \succ C_6 \succ C_5$) as the existing method (Sergi & Sari, 2021) for all values of λ. However, the weights were almost similar when the value of λ was 1.0. For the case of $\lambda = 1.0$, the reliability part was completely omitted from the Z-numbers, thus reducing the model to a regular fuzzy AHP, which is unable to handle the partially reliable decision information.

When the weights obtained from the proposed method were integrated with the Z-WASPAS model from Sergi and Sari (2021), the same ranking of alternatives was obtained. Table 14 below displays the sensitivity analysis results when the parameter from the utility function, which controls the weightage of the weighted sum and weighted product models, was changed from 0 to 1.

Table 14
Sensitivity analysis for the Z-WASPAS model

Alter-native	**Parameter in utility function**										
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
A_1	0.212	0.212	0.211	0.211	0.211	0.209	0.209	0.208	0.208	0.207	0.207
A_2	0.123	0.123	0.124	0.124	0.124	0.125	0.125	0.126	0.126	0.127	0.127
A_3	0.154	0.153	0.153	0.153	0.153	0.152	0.152	0.152	0.152	0.151	0.151
A_4	0.155	0.155	0.155	0.156	0.156	0.156	0.156	0.157	0.157	0.157	0.157
A_5	0.187	0.187	0.186	0.186	0.186	0.185	0.185	0.184	0.184	0.184	0.183
A_6	0.169	0.170	0.170	0.171	0.171	0.172	0.172	0.173	0.173	0.174	0.174

In reference to Table 14, the weights obtained from the proposed method produced a consistent ranking of alternatives ($A_1 \succ A_5 \succ A_6 \succ A_4 \succ A_3 \succ A_2$) when applied to the Z-WASPAS model from Sergi and Sari (2021).Therefore, this sensitivity analysis has shown that the proposed method is applicable for determining the weights of criteria for the application in multi-criteria decision-making problems.

Both the Z-AHP model from Sergi and Sari (2021) and the current study implemented Buckley's fuzzy AHP in which the geometric mean to aggregate the pairwise comparison matrix was used. However, the AHP model proposed by Sergi and Sari (2021) transformed Z-numbers into regular fuzzy numbers by defuzzifying the reliability parts of the Z-numbers using a defuzzification formula defined by Tüysüz and Kahraman (2020). The defuzzified reliability parts were then added to the restriction parts. It should be

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noted that the transformation of Z-numbers into regular fuzzy numbers has caused a significant loss of information (Abdullahi et al., 2020; Gardashova, 2019; Shen & Wang, 2018), especially since the original decision information in the form of Z-numbers was not preserved. On the other hand, the proposed AHP in the current study keeps the Z-numbers in the form of paired fuzzy numbers, which are mainly in their original form, except for the quantification of their magnitudes to represent the criteria weights.

Therefore, the implementation of the proposed AHP method in this study has addressed the issue of information loss since the nature of the expert's preferences is kept as the restriction and reliability components representing the linguistic evaluation of criteria. The magnitude of Z-numbers was also integrated to determine the final priority weights, which combines the restriction and reliability parts in the later step instead of converting Z-numbers into regular fuzzy numbers from the beginning. Moreover, the determination of criteria weight is important since it affects the final ranking of alternatives. In fact, the ranking of alternatives will not be affected as much when a sensitivity analysis is performed and consistent criteria weights are obtained. The criteria weights obtained using the proposed method were embedded in the Z-WASPAS model from Sergi and Sari (2021) to produce a better ranking of alternatives. Table 15 displays the final score values of Z-AHP-WASPAS (Sergi & Sari, 2021) when each criterion weight was increased by 40%.

Table 15
Sensitivity analysis for the Z-AHP-WASPAS model (Sergi & Sari, 2021)

Alter-	**Criterion with increased weight by 40%**											
native	C_1		C_2		C_3		C_4		C_5		C_6	
A_1	0.209	1	0.204	1	0.205	1	0.211	1	0.207	1	0.206	1
A_2	0.125	6	0.131	6	0.121	6	0.135	6	0.128	6	0.128	6
A_3	0.151	5	0.148	5	0.152	5	0.156	4	0.152	5	0.152	5
A_4	0.160	4	0.162	4	0.171	4	0.155	5	0.161	4	0.160	4
A_5	0.180	2	0.182	2	0.172	3	0.179	2	0.179	2	0.180	2
A_6	0.176	3	0.173	3	0.180	2	0.165	3	0.172	3	0.173	3

The ranking of alternatives was changed when the weights of C_3 and C_4 were increased by 40%, making the performance of the Z-AHP-WASPAS model from Sergi and Sari (2021) 66.67%. The performance increased to 83.33% when the Z-WASPAS model (Sergi & Sari, 2021) was integrated with the proposed Z-AHP model, which utilized the magnitude of Z-numbers in determining the criteria weights instead of conversion into regular fuzzy numbers. The integrated model maintains its consistency, except when the weight of C_4 was increased by 40%. The score values for the integrated model are shown in Table 16.

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Table 16
Sensitivity analysis for the proposed Z-AHP and Z-WASPAS models (Sergi & Sari, 2021)

Alter-native	**Criterion with increased weight by 40%**											
	C_1		C_2		C_3		C_4		C_5		C_6	
A_1	0.212	1	0.206	1	0.208	1	0.212	1	0.211	1	0.208	1
A_2	0.121	6	0.128	6	0.121	6	0.131	6	0.124	6	0.125	6
A_3	0.151	5	0.149	5	0.153	5	0.155	4	0.153	5	0.154	5
A_4	0.154	4	0.157	4	0.163	4	0.152	5	0.157	4	0.154	4
A_5	0.186	2	0.188	2	0.179	2	0.185	2	0.185	2	0.187	2
A_6	0.176	3	0.172	3	0.176	3	0.165	3	0.170	3	0.172	3

7. Conclusion

The implementation of Z-numbers in any MCDM method must consider the preservation of the restriction and reliability components to avoid the loss of decision information. The magnitude of Z-numbers was integrated with the AHP to produce a consistent criteria ranking. In the proposed model, the Z-numbers were not converted into regular fuzzy numbers because the transformation causes a loss of information. Hence, the restriction and reliability components of Z-numbers were combined using the magnitude formula to determine the priority weights. This method not only preserves the initial information in the form of Z-numbers, but also simplifies the calculation involving Z-numbers since the meaning of the magnitude of fuzzy numbers is visual and natural. However, since this study is limited to criteria ranking using the proposed AHP model, there is a need to integrate the AHP model with other MCDM methods such as TOPSIS or VIKOR to assist decision-makers in ranking the alternatives. It is important to preserve the Z-numbers when integrating these MCDM methods so that the loss of information can be avoided.

REFERENCES

Abbasbandy, S., & Hajjari, T. (2009). A new approach for ranking of trapezoidal fuzzy numbers. *Computers and Mathematics with Applications*, *57*(3), 413–419. Doi: https://doi.org/10.1016/j.camwa.2008.10.090

Abdullahi, M., Ahmad, T., & Ramachandran, V. (2020). A review on some arithmetic concepts of Z-number and its application to real-world problems. *International Journal of Information Technology and Decision Making*, *19*(4), 1091–1122. Doi: https://doi.org/10.1142/S0219622020300025

Aliev, R. A. (2013). *Fundamentals of the Fuzzy Logic-Based Generalized Theory of Decisions*. Springer-Verlag.

Aliev, R. A., Alizadeh, A. V. V., & Huseynov, O. H. H. (2015). The arithmetic of discrete Z-numbers. *Information Sciences*, *290*(C), 134–155. Doi: https://doi.org/10.1016/j.ins.2014.08.024

Aliev, R. A., Guirimov, B. G., Huseynov, O. H., & Aliyev, R. R. (2021). A consistency-driven approach to construction of Z-number-valued pairwise comparison matrices. *Iranian Journal of Fuzzy Systems*, *18*(4), 37–49.

Azadeh, A., Saberi, M., Atashbar, N. Z., Chang, E., & Pazhoheshfar, P. (2013). Z-AHP: A Z-number extension of fuzzy analytical hierarchy process. *IEEE International Conference on Digital Ecosystems and Technologies*, 141–147. Doi: https://doi.org/10.1109/DEST.2013.6611344

Buckley, J. J. (1985). Fuzzy hierarchical analysis. *Fuzzy Sets and Systems*, *17*(3), 233–247. Doi: https://doi.org/10.1016/0165-0114(85)90090-9

Buckley, James J., Feuring, T., & Hayashi, Y. (2001). Fuzzy hierarchical analysis revisited. *European Journal of Operational Research*, *129*(1), 48–64. Doi: https://doi.org/10.1016/S0377-2217(99)00405-1

Chang, D.-Y. (1996). Applications of the extent analysis method on fuzzy AHP. *European Journal of Operational Research*, *95*(3), 649–655. Doi: https://doi.org/10.1016/0377-2217(95)00300-2

Emrouznejad, A., & Ho, W. (2017). *Fuzzy Analytic Hierarchy Process*. Chapman and Hall/CRC. Doi: https://doi.org/10.1201/9781315369884

Farzam, M., Kermani, M. A., Allahviranloo, T., & Belaghi, M. J. S. (2021). A new method for ranking of Z-numbers based on magnitude value. In *Progress in Intelligent Decision Science. IDS 2020. Advances in Intelligent Systems and Computing* (pp. 841–850). Springer, Cham. Doi: https://doi.org/10.1007/978-3-030-66501-2_68

Gardashova, L. A. (2019). Z-number based TOPSIS method in multi-criteria decision making. In *Advances in Intelligent Systems and Computing* (Vol. 896). Springer International Publishing. Doi: https://doi.org/10.1007/978-3-030-04164-9_10

Kang, B., Wei, D., Li, Y., & Deng, Y. (2012). A method of converting Z-number to classical fuzzy number. *Journal of Information and Computational Science*, *9*(3), 703–709.

Leung, L. C., & Cao, D. (2000). On consistency and ranking of alternatives in fuzzy AHP. *European Journal of Operational Research*, *124*(1), 102–113. https://doi.org/10.1016/S0377-2217(99)00118-6

Perote-Peña, J., & Piggins, A. (2007). Strategy-proof fuzzy aggregation rules. *Journal of Mathematical Economics*, *43*(5), 564–580. Doi: https://doi.org/10.1016/j.jmateco.2006.09.010

Ruoning, X., & Xiaoyan, Z. (1992). Extensions of the analytic hierarchy process in fuzzy environment. *Fuzzy Sets and Systems*, *52*(3), 251–257. Doi: https://doi.org/10.1016/0165-0114(92)90236-W

Saaty, T. L. (1980). *The Analytic Hierarchy Process*. McGraw-Hill.

Sergi, D., & Sari, U. I. (2021). Prioritization of public services for digitalization using fuzzy Z-AHP and fuzzy Z-WASPAS. *Complex & Intelligent Systems*, *7*(2), 841–856. Doi: https://doi.org/10.1007/s40747-020-00239-z

Shen, K. W., & Wang, J. Q. (2018). Z-VIKOR method based on a new comprehensive weighted distance measure of Z-number and its application. *IEEE Transactions on Fuzzy Systems*, *26*(6), 3232–3245. Doi: https://doi.org/10.1109/TFUZZ.2018.2816581

Tüysüz, N., & Kahraman, C. (2020). CODAS method using Z-fuzzy numbers. *Journal of Intelligent & Fuzzy Systems*, *38*(2), 1649–1662. Doi: https://doi.org/10.3233/JIFS-182733

Vaidya, O. S., & Kumar, S. (2006). Analytic hierarchy process: An overview of applications. *European Journal of Operational Research*, *169*(1), 1–29. Doi: https://doi.org/10.1016/j.ejor.2004.04.028

van Laarhoven, P. J. M., & Pedrycz, W. (1983). A fuzzy extension of Saaty’s priority theory. *Fuzzy Sets and Systems*, *11*(1–3), 229–241. Doi: https://doi.org/10.1016/S0165-0114(83)80082-7

Voskoglou, M. (2019). Application of fuzzy numbers to assessment processes. In K.-P. Mehdi (Ed.), *Encyclopedia of Information Science and Technology* (pp. 407–420). IGI Global. Doi: https://doi.org/10.4018/978-1-5225-7368-5.ch030

Wang, Y. M., Elhag, T. M. S., & Hua, Z. (2006). A modified fuzzy logarithmic least squares method for fuzzy analytic hierarchy process. *Fuzzy Sets and Systems*, *157*(23), 3055–3071. Doi: https://doi.org/10.1016/j.fss.2006.08.010

Wang, Y. M., Luo, Y., & Hua, Z. (2008). On the extent analysis method for fuzzy AHP and its applications. *European Journal of Operational Research*, *186*(2), 735–747. Doi: https://doi.org/10.1016/j.ejor.2007.01.050

Xu, R. (2000). Fuzzy least-squares priority method in the analytic hierarchy process. *Fuzzy Sets and Systems*, *112*(3), 395–404. Doi: https://doi.org/10.1016/S0165-0114(97)00376-X

Zadeh, L. A. (1965). Fuzzy sets. *Information and Control*, *8*(3), 338–353. Doi: https://doi.org/10.1016/S0019-9958(65)90241-X

Zadeh, L. A. (2011). A Note on Z-numbers. *Information Sciences*, *181*(14), 2923–2932. Doi: https://doi.org/10.1016/j.ins.2011.02.022

Zeinalova, L. M. (2018). A Z-number valued analytical hierarchy process. *Chemical Technology, Control and Management*, *2018*(3), 88–94. Doi: https://doi.org/https://doi.org/10.34920/2018.4-5.88-94

Zhang, X., Ma, W., & Chen, L. (2014). New similarity of triangular fuzzy number and its application. *The Scientific World Journal*, *2014*. Doi: https://doi.org/10.1155/2014/215047